SATELLITE IMAGE DENOISING USING LINEAR WAVELETS

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Abstract—Modeling of two major linear wavelet denoising techniques; Poisson Unbiased Risk Estimate Linear Expansion of Thresholds (PURELET) and Stein’s Unbiased Risk Estimate Linear Expansion of Thresholds (SURELET) in application to satellite images is presented in this paper. Satellite image sensors are often corrupted by noise that is effectively enhanced by estimation of noise and robust denoising algorithm application. By dynamic thresholding of unbiased estimates, optimized enhancement of spatial resolution can be obtained. Poisson unbiased risk estimate (PURE) is an unbiased estimator of the mean square error between the original and estimated images. Stein’s unbiased risk estimate (SURE) is the apriori estimation of Mean Square Error resulting from an arbitrary processing of noisy data for the additive Gaussian noise model. By minimizing the unbiased estimates using Linear Expansion of Threshold (LET), noise can be significantly reduced and resolution can be enhanced.

Keywords: PURE, SURE, LET, Denoising, Satellite Images

I. INTRODUCTION

In satellite imaging, images captured from sensors are often expressed as the pixel value at a particular location in terms of the number of photons received by the corresponding capture at a particular instant of time. When the image pixel density is low, the available photons will be limited in number. Most of the noise arises from the fluctuation of the number of incoming photons, but additional perturbations are generated by the thermal instabilities of the electronic acquisition devices and the analog-to-digital conversion. There are two main approaches to deal with degradations. The first one is to develop analysis tools that will be robust with respect to noise and the second one is pre-processing step that will denoise the data. Denoising is a computationally best optimization process where huge multidimensional datasets are produced by standard satellite image modalities with less calls for less generic statistical measurement model. The general solution of Poisson noise is “Gaussianizing” this is carried out by non linear data that is applied to the raw data. It has been theorized by anscombe and applied first by donoho [8]. In Median filtering, transform domain methods such as Fourier and discrete Fourier transform will also be employed for denoising the medical images. But they introduce blur in the images. This will damage the texture in image. Wavelet based methods are widely used in medical image denoising and disease diagnosis. Many of these algorithms are related to shrinkage/thresholding to wavelet image coefficients. The difficult task in this is to select appropriate threshold selection [7]. The unnormalized Haar transform is appropriate for Poisson noise in the view of the fact that it is self-reproducing across scales. By capturing this advantage [8] consequently the Bayesian intensity estimate for multiscale multiplicative innovation model is applied. Multiscale analysis is a powerful tool in denoising procedures [9]. Non local Means Algorithm which is introduced by Buades [10]. So far, algorithm was experimented exclusive for medical imaging. In this paper exploration of how well these algorithms fit in best for satellite imaging is done. The prime objective of enhancing resolution of a satellite image is to increase spatial resolution by reducing noise incurred by the sensor. Increasing the spatial resolution increases the pixel density at the cost of addition of noise during the reconstruction process. The spatial resolution is dependent on the IFOV (Instantaneous Field Of View) which is the measure of ground area sensed at given instant of time based on the geometric properties of the imaging system. The dimension of the ground projected is given by IFOV, which is dependent on the altitude and the viewing angle of sensor. The finer the IFOV, the higher the spatial resolution. Spatial resolution is often prone to blurring of the image, due to improper focusing, atmospheric scattering and target motion. The amount of noise actually depends on the signal intensity, which is often modeled as an array of additive independent (typically Gaussian) random variable, especially when the magnitude of the measured signal is sufficiently high. Imaging sensors have a certain SNR based on their design. The energy reflected by the target must have a signal level large enough for the target to be detected by the sensor. The signal level of the reflected energy increases if the signal is collected over a larger IFOV or if it is collected over a broader spectral bandwidth. Collecting energy over a larger IFOV reduces the spatial resolution while collecting it over a larger bandwidth reduces its spectral resolution. Noise originating from the sensors often modeled in either Gaussian or Poisson noise model for ease of estimation. For the Poisson noise model, an unbiased estimate for the mean square error (MSE) between the original image and denoised image has been recently proposed which is quite similar to the well-known Stein’s unbiased risk...
estimate (SURE) for the additive Gaussian noise model [6]. The Poisson unbiased risk estimate (PURE) is defined in the unnormalized Haar wavelet domain. Furthermore, the proposed algorithms used to denoise the Poisson images by minimizing PURE, thereby minimizing the MSE. Since the MSE is inversely proportional to the peak signal-to-noise ratio (PSNR), minimizing PURE has the effect of maximizing the PSNR. Meanwhile, recent developments have shown that the wavelet transform does not optimally represent the discontinuities in images, since the wavelet functions are isotropic and possess very limited directional capabilities (a wavelet has directional vanishing moments (DVMs) only along the horizontal and vertical directions [5]). This limits the performance of wavelet-based denoising algorithms. If an alternative transform using spatially anisotropic basis functions with more directional capabilities is used to define PURE, then we must be able achieve better denoising performance in terms of visual quality as well as the PSNR. Conventionally these are tested for medical and normal imaging applications. These concepts widely cater for satellite imaging applications. In this paper, a comparative analysis is made between two biased estimates and optimizes how well it fit on to satellite image applications especially for panchromatic images that are captured from a narrow spectral range of 0.40-0.70 micrometers by satellite image sensor.

II. SURE

Stein’s unbiased risk estimate (SURE) is an a priori estimation of the MSE resulting from an arbitrary processing of noisy data. Stein’s unbiased risk estimate (SURE) is an unbiased nonlinear estimator of the mean-squared error. Stein’s unbiased risk estimate directly parameterizes the denoising process as a sum of elementary nonlinear processes with unknown weights. After the estimation of the mean square error between the test image and the denoised image, minimization of unbiased estimate is performed. In other words, it provides an indication of the accuracy of a given estimator. This is important as the true mean-squared error of an estimator is a function of the unknown parameter to be estimated, and thus cannot be determined exactly. The denoising approach amounts to minimizing over a range of reasonable denoising functions that Minimizes MSE over the same range of functions, up to a small random error inversely proportional to the square root of the number of samples. By proper thresholding amount of noise can be significantly minimized. Soft thresholding is generally preferred for denoising applications.

Soft-thresholding function defined by

\[ \Theta(y) = \text{sign}(y)(|y| - T) \tag{2.1} \]

In denoising applications, the performance is often measured in terms of peak signal-to-noise ratio (PSNR), which can be defined as follows

\[ \text{PSNR} = 10 \log_{10} \left( \frac{\max(y)}{\sigma^2} \right) \tag{2.2} \]

The objective of image denoising is to maximize the PSNR which results in minimization of MSE

\[ \text{MSE} = \| \Theta(y) - x \|^2 = \Theta(y)^2 - 2 \Theta(y)x + x^2 \tag{2.3} \]

Where \( Y = x + b \).

Let \( \mu \in \mathbb{R}^d \) be an unknown parameter and let \( x \in \mathbb{R}^d \) be a measurement vector whose components are independent and distributed normally with mean \( \mu \) and variance \( \sigma^2 \). Suppose \( h(x) \) is an estimator of \( \mu \) from \( x \), and can be written, \( h(x) = x + g(x) \)

Stein’s unbiased risk estimate is given by

\[ \text{SURE}(h) = d \sigma^2 + \|g(x)\|^2 + 2 \sigma^2 \sum_{i=1}^{d} \frac{\partial}{\partial x} g_i(x) \tag{2.4} \]

where \( g_i(x) \) is the \( i \)th component of the function \( g(x) \), and is the Euclidean norm. The importance of SURE is that it is an unbiased estimate of the mean-squared error (or squared error risk) of \( h(x) \), i.e. \( \mathbb{E}_\mu \{ \text{SURE}(h) \} = \text{MSE}(h) \), with \( \text{MSE}(h) = \mathbb{E}_\mu \| h(x) - \mu \|^2 \)

Minimizing SURE will act as a catalyst for minimizing the MSE. There is no dependence on the unknown parameter \( \mu \) in the expression for SURE above. Thus, it can be manipulated to determine optimal estimation without knowledge of \( \mu \). Transform-domain denoising consists in first defining a complementary pair of linear transformation operators \( D \) (decomposition) and \( R \) (reconstruction) such that \( RD = \text{Id} \); typically, \( D \) is a bank of decimated or undecimated filters. Once the size of the input and output data are frozen, these linear operators are characterized by matrices, respectively

\[ D = [d_{ij}]_{(i,j) \in [1,L] \times [1,N]} \tag{2.6} \]

\[ R = [r_{ij}]_{(i,j) \in [1,N] \times [1:L]} \]

that satisfy the perfect reconstruction property \( RD = \text{Id} \). Here 2 D real transforms is considered, i.e. \( d_{ij}, r_{ij} \in \mathbb{R} \).

The whole denoising process then boils down to the following steps:

1. Apply \( D \) to the noisy signal \( y = x + b \) to get the transformed noisy coefficients \( w = Dy = [w_{i}]_{i \in [1:L]} \)

2. Apply a (possibly multivariate) thresholding function \( \Theta(w) = |\Theta(w)|i \in [1:L] \)

3. Revert to the original domain by applying \( R \) to the thresholded coefficients \( \Theta(w) \), yielding the denoised estimate \( \hat{x} = R\Theta(w) \).

Such a denoising procedure can be summarized as a function of the noisy input coefficients:

\[ \hat{x} = F(y) = R\Theta(Dy) \tag{2.8} \]
The SURE-LET strategy consists in expressing F as a linear expansion of denoising algorithms F_k, according to:

\[ F(y) = \sum_{k=1}^{K} R \phi_k(Dy) \]  

(2.9)

where \( \phi_k(\cdot) \) are elementary (possibly multivariate) thresholding functions and \( F_k(y) = R \phi_k(Dy) \)

A periodic boundary extension implementation of this structure gives rise to decomposition and reconstruction matrices D and R made of J circulant submatrices (i.e. diagonalized with an N-point DFT matrix) \( D_i \) and \( R_i \) of size \( N \times N \) each, with coefficients:

\[
\begin{align*}
\hat{G}_1(z^{-1}) & \quad \hat{G}_1(z) \\
\hat{G}_2(z^{-1}) & \quad \hat{G}_2(z) \\
\vdots & \quad \vdots \\
\hat{G}_J(z^{-1}) & \quad \hat{G}_J(z)
\end{align*}
\]

Fig.2.1 Surelet - Filter structure

Where \( G_i(z) = \sum_{n \in \mathbb{Z}} g_i[n]z^{-n} \) and J synthesis filters \( G_i(z) = \sum_{n \in \mathbb{Z}} g_i[n]z^{-n} \). The decomposition and reconstruction can be expressed as

\[
\begin{align*}
[D \theta_{kl}] & = \sum_{n} G[k-l+nN] \\
[R \theta_{kl}] & = \sum_{n} G[k-l+nN]
\end{align*}
\]

(2.10)

\[ M = [F_k(y)^T F_k(y)]_{(k,1)-(k,J)} \]

C=[\int \hat{F}_k(y)^T (\text{Id}-R_{k1} D_{k1}) y^2 \sigma^2 \text{div} \{ F_k(y) \}]_{(k,1)-(k,J)}

4. Compute the matrix M given in above equation and deduce the optimal (in the minimum SURE sense) linear parameters \( a_{j,k} \).

5. The noise-free image \( \hat{x} \) is finally estimated by the sum of each \( F_k \) weighted by its corresponding SURE-optimized \( a_{j,k} \) and the reconstructed lowpass residual subband is obtained.

IV. PURE

PURE (Poisson unbiased risk estimate) is defined in the unnormalized Haar wavelet domain by making use of the following property: in every scale, the approximation coefficients of the unnormalized Haar wavelet transform of a Poisson image are also Poisson distributed and hence the detail coefficients are differences of Poisson random variables. In order to minimize, PURE is expressed the linear expansion of thresholds (LET). The overall procedure reduces to inverting a \( K \times K \) matrix (\( K \) may be as small as six), resulting in a computationally efficient and flexible yet robust denoising algorithm known as PURE-LET [2].

In the AWGN channel model, it is possible to find the processing \( F_{\text{opt}} \) that minimizes the expected MSE/PURE, without an explicit knowledge of the PDF of the original noise-free signal \( p(x) \). We denote the joint density of \( x \) and \( y \) by:

\[ p(x, y) = p(y|x)p(x) \]  

(3.1)

The marginal PDF of the noisy observation is

\[ r(y) = R_N \int p(x, y) dx \]  

(3.2)

The minimum expected MSE/PURE estimate of the intensity of a Poisson process embedded in an AWGN of variance \( \sigma^2 \) is given by:

\[ F_{\text{opt}}(y) = [y + \sqrt{r(y) + \text{var}(y - y + \text{var}(y - \text{var}(y) - r(y) \text{var}(y - \text{var}(y)) \text{div} \{ F_{\text{opt}}(y) \})}]_{(y)} 
\]

(3.3)

The optimal search for the minimum PURE vector of parameters \( a = [a_1, a_2, \ldots, a_k] \) that reduces to the solution of the following linear system of K equation

\[ \sum_{k=1}^{K} F_k(y)^T \hat{F}_k(y) a_{k} = y^T \hat{F}(y) - \sigma^2 \text{div} \{ F_{\text{opt}}(y) \} \]  

(3.4)

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V. PURE AS UNNORMALIZED HAAR WAVELET TRANSFORM

The analysis of filter can be given in z transform by
\[ H_a(z) = 1 + z^{-1}, \quad G_a(z) = 1 - z^{-1} \]
and the corresponding synthesis pair is \( H_s(z) = H_a(z^{-1})/2, \quad G_s(z) = G_a(z^{-1})/2 \). The unnormalized Haar scaling coefficients of the measurement \( y = z + b \) at scales \( j = 1, \ldots, J \) are denoted by \( s_j \in \mathbb{R}^N_j \), where \( N_j = N/2^j \), and \( d_j \in \mathbb{R}^N_j \) stands for the associated wavelet coefficients (we assume that the signal dimension is divisible by \( 2^J \)).

By setting \( S_0 = y \), these coefficients are obtained from the following sums and differences:
\[
\begin{align*}
    s_j &= s_{2j} + s_{2j+1} \quad \text{for } j = 1, \ldots, J \\
    d_j &= s_{2j} - s_{2j+1} \quad \text{for } j = 1, \ldots, J
\end{align*}
\]

The key properties of the unnormalized Haar DWT are the following.

1. It is an orthogonal transform. In particular, we can split the MSE into subband specific error terms:
\[
\text{MSE} = \frac{1}{N} \sum_{j=1}^{J} \left( \sum_{i=1}^{\frac{N}{2^j}} |s_i - d_i|^2 \right)
\]
(3.7)
This implies that we can minimize the MSE for each subband independently, while ensuring a global signal-domain MSE minimization.

2. At a given scale \( j \), the scaling coefficients of an input vector of independent Poisson random variables are also independent Poisson random variables, because the sum of independent Poisson random variables is also a Poisson random variable with intensity equal to the summed intensities.

VI. RESULTS AND DISCUSSIONS:

The Panchromatic images of size \([1024*1024]\) are subjected for image under test. The objective is to compare two unbiased estimates of two different noise models. Corresponding Computational time is also estimated.
From the results, it has been found that for the same input PSNR, two unbiased estimates provide difference in PSNR values with the same computational time. By minimizing PURE, noise can be reduced. PURE was originally defined in the Haar wavelet domain. Since the wavelet functions are isotropic in every scale and have limited directional capabilities, the denoising performance obtained by minimizing PURE is poor around the discontinuities in the images. Perceptually the image obtained by PURELET drastically reduced in resolution. For SURELET, PSNR is improved compared to PURELET. The objective is to maximize the PSNR retaining lesser MSE. Here PSNR is considered as the prime factor which parameterizes the energy compaction at a particular pixel density. The maximum PSNR is retained for SURELET than for PURELET.

CONCLUSION

From the results, it is observed that SURELET has the highest PSNR value over PURELET. By optimum threshold variations in SURELET, PSNR can still be improved. Extension of PURELET to the directionlet functions would possess spatial anisotropy and better directional capabilities with improved PSNR values. SURELET provides visually appealing denoised results. Thus SURELET outperforms PURELET rendering better resolution modalities and preserves maximum photons at a particular pixel density.

REFERENCES