OPTIMAL CONTROL OF INVERTED PENDULUM BASED ON TWO PID AND LQR ARRANGEMENT AND IMPROVED BP NEURAL NETWORK

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Abstract— BP Neural Network has a longer training time and a slow convergence. To deal with the defects of BP Neural Network a modified BP algorithm is proposed in the paper. The algorithm is applied for the control of Inverted Pendulum, a highly non linear system inherently being open loop unstable. Levenberg-Marquardt algorithm is used for the training purpose. The training samples are being collected by a two PID and one LQR arrangement. The simulation results prove that the modified BP algorithm for inverted pendulum control gives better efficiency, lesser training time and faster convergence.

Keywords— Artificial Neural networks, Inverted Pendulum, Modified Back Propagation Algorithm, PID, LQR

I. INTRODUCTION

Inverted Pendulum is considered to be an ideal experiment for the experimentation and scientific research of different control theories as it has a typical non linear dynamics and is inherently uncertain and open loop unstable. The IP system is of higher order, time varying, multivariable and highly coupled system making it one of the most important and interesting control engineering problems.

The problem involves a cart, able to move in backward and forward directions, and a pendulum hinged to the cart at the bottom of its length. The pendulum is free to move in the same plane as the cart. The control problem is to stabilize the pendulum in its upright position. Realistically, this system can be seen as a representation of altitude control problems where the aim is to maintain a desired vertical position.

Artificial Neural Network (ANN) with their ability to learn, classify patterns and approximate functions and their potential for parallel hardware implementation makes it capable to meet the demands of control requirements in increasingly complex dynamical control systems. Artificial Neural Network becomes to be more and more widely applied in regression analysis, pattern and sequence recognition, data processing, system identification and control, intelligent control and various other areas. Artificial Neural Network is flexible, adaptive and self organized and is capable of greater fault tolerance than a classical network [1]-[2].

In this paper, a modified Back Propagation Neural Network algorithm is proposed to control linear Inverted Pendulum. The training samples are being collected with the help of two PID and one LQR arrangement. The algorithm reduces the training time of neural networks and achieves faster convergence, higher accuracy and better control effect. The control of Inverted Pendulum is being compared with the classical control by PID. It has also been shown with the results that Neural Network add artificial intelligence to the system by handling uncertain deviations in better way than PID.

The paper is organized in 6 sections. Section I represents the relevance and general introduction of the paper. Section II describes the mathematical model of Linear Inverted Pendulum, section III describes the artificial neural network algorithm, section IV represents the Artificial Neural Network controller training and design, Section V results are shown and in Section VI a conclusion is presented. At the end, a brief list of references is given.

II. MATHEMATICAL MODEL OF INVERTED PENDULUM

The linear inverted pendulum of Googol Technology Ltd. is chosen as the control object for the experiment [3]. The free body diagram of pendulum mounted on a motor driven cart is shown in Fig. 1. Let $M$ denote cart mass, $m$ rod mass, $b$ friction coefficient of the cart, $l$ distance from rod rotation axis center to rod mass center, $I$ rod inertia, $F$ force acting on the cart, and $x$ cart position and $\theta$ is the angle between rod and vertically downward position.

![Fig. 1 Free body diagram of pendulum mounted on cart](image-url)
$N$ and $P$ are the interactive forces of cart and rod in the horizontal and vertical directions respectively.

Considering horizontal direction, summing all the forces on the cart we get following equation:

$$M \ddot{x} + b \dot{x} + N = F$$  
(1)

Due to moment of pendulum the force exerted in the horizontal direction is:

$$N = m \frac{d^2}{dt^2}(x + l \sin \theta)$$  
(2)

Rewriting it as:

$$N = m \ddot{x} + ml \dot{\theta} \cos \theta - ml^2 \dot{\theta}^2 \sin \theta$$  
(3)

Substituting this equation into the first equation, first equation of motion is obtained:

$$(M + ml) \ddot{x} + b \dot{x} + ml \dot{\theta} \cos \theta - ml^2 \dot{\theta}^2 \sin \theta = F$$  
(4)

Now summing the forces perpendicular to the pendulum:

$$P \sin \theta + N \cos \theta - mg \sin \theta = ml \ddot{\theta} + m \ddot{x} \cos \theta$$  
(5)

To remove $P$ and $N$ terms in the equation above, by moment conservation we get the following equation:

$$-P \sin \theta - N \cos \theta = J \ddot{\theta}$$  
(6)

Combining these last two equations, we get the second dynamic equation:

$$(I + ml^2) \ddot{\theta} + mgl \sin \theta = -ml \cos \theta$$  
(7)

Hence the set of equations completely defining the dynamics of the inverted pendulum are:

$$(M + ml) \ddot{x} + b \dot{x} + ml \dot{\theta} \cos \theta - ml^2 \dot{\theta}^2 \sin \theta = F$$  
(8)

$$(I + ml^2) \ddot{\theta} + mgl \sin \theta = -ml \cos \theta$$  
(9)

The values for the experiment is taken as $M = 1.096$ kg, $m = 0.109$ kg, $b = 0.1$ N/m/sec, $l = 0.25$ m, $I = 0.0034$ kg*m$^2$.

### III. ARTIFICIAL NEURAL NETWORK ALGORITHM

In past years, a number of ANN models have been developed and studied in depth, most of them have Back Propagation (BP) feed forward network as the most essential part of the network[4]-[7]. As the name implies, the error propagate backwards from the output nodes to the inner nodes. Standard BP algorithm works on gradient descent method. The network tries to adapt its weights in the fashion that the error is minimized. The algorithm is as below:

```plaintext
initialize all the weights in the network (start with some small random values)
do for each sample error in the training set
    O = neural net output
    T = teacher output or desired output
    calculate error (T - O) at the output units
compute delta_wh for all weights from hidden layer to output layer
compute delta_wi for all weights from input layer to hidden layer
update the weights in the network
until all samples classified correctly or stopping criterion satisfied
return the network
```

Although Back Propagation (BP) Algorithm is most widely used algorithm but it has its own limitations. BP has a longer training time, and sometimes it may fall into local minima, making BP algorithm unsuitable in many real time applications. A lot of development has been done to make the training time faster such as adding an additional momentum, adaptive learning rate method, conjugate gradient method, Quasi Newton method, Levenberg-Marquardt method. In this paper LM algorithm is used for training as it has the fastest convergence and higher accuracy compared with all other algorithms. LM algorithm interpolates between Gauss-Newton algorithm and the method of gradient descent. LM algorithm even if have started off very far from the final minimum finds a solution in many cases.

The formula for weight optimization and threshold updation in LM algorithm is as follows:

$$X_{k+1} = X_k - (J^T J + \mu I)^{-1} J^T e$$  
(10)

Where $J$ is the Jacobian matrix from the differential of error to weight value, $e$ denotes the error vector and $\mu$ is a scalar. When $\mu$ tends to zero the method approaches towards Newton method and when $\mu$ tends to infinite it approaches towards the well known Steepest descent method [8].

### IV. ARTIFICIAL NEURAL NETWORK CONTROLLER TRAINING AND DESIGN

#### A. Selection of training samples with two PID and one LQR arrangement

The efficiency of any Neural Network lies in its training. For more number of training samples, the accuracy is increased but the drawback is that training time and computational complexity increases. With lesser number of samples, the information may be lost[9]. Hence it is very important to select a reasonable number of samples for training. In actual simulation, we made an arrangement of two PID controllers one for position and other for angle and one LQR(linear quadratic Regulator) in the feedback. The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control[10]. Tuning of PID controllers is done to make the angle stable at upright position. The simulink model for the arrangement is shown in Fig. 2.
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After tuning the parameter values are:

For position PID: \( K_p = 30 \), \( K_i = 30 \), \( K_d = 25 \)

For angle PID: \( K_p = 60 \), \( K_i = 20 \), \( K_d = 15 \)

LQR parameter values:

\[ K_x = 31.6228 \]
\[ K_{\dot{x}} = 16.4046 \]
\[ K_\theta = -50.2514 \]
\[ K_{\dot{\theta}} = 1.5 \]

The simulation is done and the samples are collected. At next step, one by one discrete impulses of different angles are given as disturbance to the arrangement and samples are collected after the tuning of PID. The discrete impulse are given in the range of \((-30^\circ \text{ to } +30^\circ)\) with the interval of \(5^\circ\) in between. The simulink model for the arrangement with external disturbance is shown in Fig. 3.

For efficient training, the redundant samples from different sets are deleted and then combined to make a matrix of Input and Output. It has been found that with around 1000 training samples, the training results were more reasonable.

Hence the basic approach for design is to start with two layers one hidden and one output and then varying the number of hidden layers and neurons in the hidden layer and checking for the network performance. After a number of trials the network that gave efficient control has the following specifications.

The neural network designed has two inputs: position error & angle error and has one output \( F \). The network has an input layer that has 2 neurons, three hidden layers that have 15, 10 and 8 neurons respectively, and an output layer that has 1 neuron. The transfer function for hidden layers is chosen as ‘tansig’ and for output layer ‘purelin’. The backpropagation network training function is chosen as ‘trainlm’ for Levenberg-Marquardt method of training.. After the network is built, the training of the neural network is done with the help of samples collected before. The learning rate for the training of network is set at 0.99, maximum epoch at 1000, and the goal is set at 0.01.

The network training code is as follows:

```matlab
net=newff(In,Out,[15,10,8],{'tansig','tansig','tansig','purelin'},'trainlm');
net.trainParam.show=100;
net.trainParam.lr=0.99;
net.trainParam.epoch=1000;
net.trainParam.goal=0.01;
net.trainParam.max_fail=30;
[net,tr]=train(net3,In,Out);
a=sim(net,In);
gensim(net);
```

After the successful training of the neural net, it has been placed in place of PID and the results are compared.

V. RESULTS

The results obtained from the PID control and the Neural Control are compared and from Fig. 4 and Fig. 5 it is found that the Neural network is able to control the angle in a better fashion with lesser overshoot and lesser settling time than the PID control.
Also if we give any disturbance from (-30° to +30°), the designed neural network is able to adapt with the disturbance and controls the angle again. In case of PID control, after applying the disturbance we have to tune the PID parameters in order to get the control. Hence, the neural network has added intelligence to the control as expected. Fig 5 shows the angle control when an external disturbance of 30° is applied.

Therefore, it can be concluded that the designed neural network is able to control the angle of Inverted Pendulum better than the PID in both the conditions with no disturbance and with disturbance applied. It has also been seen that the network is able to adapt and give efficient controlling even in the presence of external disturbance of 60° representing the adaptive quality of neural network.

CONCLUSION

In this paper a modified BP Artificial Neural Network is proposed to control the Inverted Pendulum in the upright position. The Simulation experiment has been carried out and the results show that it gives a better convergence, little overshoot and is better able to control the non linear system than classical control by PID.

REFERENCES


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