A NOVEL CONTROL ALGORITHM FOR NONLINEAR SYSTEMS

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Abstract-In the design procedure of the controller, parallel distributed compensation (PDC) scheme was utilized to construct a global fuzzy logic controller by blending all local state feedback controllers. A stability analysis was carried out for a real structure system using Lyapunov method.

Keywords-Fuzzy Lyapunov Function, Takagi-Sugeno form, Swarm Intelligence Algorithm

I. INTRODUCTION

Swarm intelligence is also a popular field, which catches many researchers’ attention. Many algorithms, which are inspired by the tiny intelligent from creatures in the Mother Nature, are included in this field. In general, the swarm intelligence method requires evolutionary computing and is imitating the particular behaviors or the survival skills of creatures. For instance, Cat Swarm Optimization (CSO) is proposed by modeling the cat’s behaviors in the mathematic forms (Chu et al., 2006; 2007); Interactive Artificial Bee Colony (IABC) is proposed by simulating the nectar collection behavior of bees (Tsai et al., 2009); and Evolved Bat Algorithm is proposed base on the prey finding process of bats (Tsai et al., 2012). These algorithms have been applied to solve many problems in engineering and finance fields.

II. THE NONLINEAR PROBLEMS

Consider a flow field within an anisotropic but homogeneous porous structure under the action of a small amplitude incident wave train. The fluid is assumed to be incompressible. The flow field can be proved to be irrotational if the induced motion is small and periodic and initially irrotational. In the flow field, a single-valued velocity potential exists and can be defined by (Lee, 1987)

\[ u_j = a_j^2 \frac{\partial \phi}{\partial x_j} \quad (2-1) \]

in which the parameter \( a_j \) is defined as follows

\[ a_j^2 = \left( \frac{i}{f_j - is_j} \right), \quad j = 1, 2 \quad (2-2) \]

where the subscript \( j \) represents the variables in \( x \)- or \( z \)-direction, respectively, as it is equal to 1 or 2, \( s \) is the inertial force coefficient given in Morrison’s equation, and \( f \) is the linear friction drag coefficient according to the hypothesis of Lorentz’s equivalent work, which is given by

\[
\begin{align*}
    f_j &= \frac{\epsilon_j}{\sigma} \left[ \frac{1}{K_{pj}} \int_{t_0}^{t_0+T} \int_{V} |u_j|^2 dtdV + \int_{t_0}^{t_0+T} \int_{V} |u_j|^3 dtdV \right] \\
    &= \frac{\epsilon_j}{\sigma} \left[ \frac{1}{K_{pj}} \int_{t_0}^{t_0+T} \int_{V} |u_j|^2 dtdV + \int_{t_0}^{t_0+T} \int_{V} |u_j|^3 dtdV \right] \\
    &= \frac{\epsilon_j}{\sigma} \left[ \frac{1}{K_{pj}} \int_{t_0}^{t_0+T} \int_{V} |u_j|^2 dtdV + \int_{t_0}^{t_0+T} \int_{V} |u_j|^3 dtdV \right] \\
\end{align*}
\]

where \( C_{fj} \) is a dimensionless turbulent coefficient, \( v \) is the kinematic viscosity of the fluid, \( \epsilon_j \) is the directional area porosity, and \( K_{pj} \) the intrinsic permeability of the porous media. For specific porous media, they can be evaluated a priori from standard tests or from empirical expressions and be applied when the wave length is long with respect to wave amplitude and media grain size (Ward, 1964; Dinoy, 1971; Sollitt and Cross, 1972).

The velocity potential corresponding to the permeable structure satisfies the modified Laplace equation which is derived from the continuity equation

\[
\sum_{j=1}^{2} \left[ a_j^2 \frac{\partial^2 \phi}{\partial x_j^2} \right] = 0 \quad (2-4)
\]

III. ANALYTICAL SOLUTIONS

3.1 Fuzzy modeling of a structure system
An overall closed-loop controlled system obtained as follow:

\[
\dot{X}(t) = \sum_{i=1}^{r} \sum_{l=1}^{l} h_i(t)h_l(t)[(A_l - B_lK_l)X(t)] + E_l\phi(t)
\]

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A typical stability condition for fuzzy system is proposed here as follows:

Theorem 1: The equilibrium point of fuzzy control systems stable in the large if there exist a common positive definite matrix \( P \) and feedback gains \( K \) such that the following two inequalities are satisfied:

\[
(A_i - B_i K_i)^T P + P(A_i - B_i K_i) + \frac{1}{\eta} P E_i E_i^T P < 0
\]

and

\[
\frac{(A_i - B_i K_i) + (A_i - B_i K_i)^T}{2} P + P \frac{(A_i - B_i K_i) + (A_i - B_i K_i)^T}{2} \frac{1}{\eta} P E_i E_i^T P < 0
\]

with \( P = P^T > 0 \), for \( i = l \leq r \) and \( i = 1, 2, \cdots, r \)

Proof: The proof is lengthy and can be derived in the similar approach of Yeh et al. (2008)

3.2 Evolved Bat Algorithm

Evolved Bat Algorithm (EBA) is proposed by Tsai et al. in 2012. It is developed base on the bat echolocation system in the natural world. Unlike other swarm intelligence algorithms, the strong point of EBA is that it only has one parameter, which is called the medium, need to be determined before employing the algorithms to solve problems. Choosing different medium determines different searching step size in the evolutionary process. In this study, we choose the air to be the medium because it is the original existence medium in the natural environment in which bats live.

The operation of EBA can be summarized in 4 steps:

Step 1. Initialization: the artificial agents are spread into the solution space by randomly assigning coordinates to them.

Step 2. Movement: the artificial agents are moved according to Eqs. (3-39)-(3-40). A random number is generated and then it is checked whether it is greater than the fixed pulse emission rate. If the result is positive, the artificial agent is moved using the random walk process.

\[
X_i^t = X_i^{t-1} + D
\]

where \( X_i^t \) indicates the coordinate of the \( i \)-th artificial agent at the \( t \)-th iteration, \( X_i^{t-1} \) represents the coordinate of the \( i \)-th artificial agent at the last iteration, and \( D \) is the moving distance that the artificial agent goes in this iteration.

\[
D = \gamma \cdot \Delta T
\]

where \( \gamma \) is a constant corresponding to the medium chosen in the experiment, and \( \Delta T \in [-1, 1] \) is a random number. \( \gamma = 0.17 \) is used in our experiment because the chosen medium is air.

\[
x_i^{best} = \beta \cdot (x_{best} - x_i^t), \beta \in [0, 1]
\]

where \( \beta \) is a random number; \( x_{best} \) indicates the coordinate of the near best solution found so far throughout all artificial agents; and \( x_i^{best} \) represents the new coordinates of the artificial agent after the operation of the random walk process.

Step 3. Evaluation: the fitness of the artificial agents is calculated by the user defined fitness function and updated to the stored near best solution.

Step 4. Termination: the termination conditions are checked in order to decide whether to go back to step 2 or terminate the program and output the near best solution.

The evaluation criterion for determining the fitness of a bat is based on an user defined fitness function. A fitness function is employed in the paper to find the common symmetric positive definite matrix and the control force of the controller.

IV. EXPERIMENT AND SIMULATION RESULT

A nonlinear system is given in this section to illustrate the control scheme and the usability of employing EBA to find system parameters for the control system. By applying the PDC scheme, the temporary states of the nonlinear system can be decomposed into different T-S fuzzy models. Hereinafter, the LMI stability condition obtained by the Lyapunov function is used to examine the stability of the designed fuzzy controller. Moreover, a fitness function is designed for utilizing EBA to find the proper system parameters. An inverted pendulum system, where \( u(t) \) denotes the control force generated by the controller and \( \Phi(t) \) is the rotational angle of the pendulum.

The system can be modeled from the dynamics:

\[
\frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 9 \gamma \end{bmatrix}
\]

where \( x \) is the radius of the pendulum vertically, \( y \) denotes the rotation velocity, and \( \gamma \) means the demand output angle. Every temporary state of the inverted pendulum system can be decomposed by fuzzy IF-THEN rules; by combining all decomposed fuzzy IF-THEN rules, the whole nonlinear system is able to be approximated. Similar schemes can be found in previous studies (see Liu and Lin, 2012; 2013; Yeh et al., 2008). Hence, the approximated inverted pendulum nonlinear system via T-S fuzzy model is decomposed as follows:

Rule 1: IF \( x \geq \frac{\pi}{3} \) (rad), THEN \( \dot{x} = A_1 \tilde{x} + B_1 u \)

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Rule 2: IF $x \approx \frac{\pi}{90}$ (rad), THEN $\dot{x} = A_2 \ddot{x} + B_1 \mu$.

where $A_1 = \begin{bmatrix} 0 & 1 \\ 9.81 & -2 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ -5 & -3 \end{bmatrix}$,

$B_1 = \begin{bmatrix} 0 \\ -10 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ -0.081 \end{bmatrix}$, and $\ddot{x} = \begin{bmatrix} \sin x \\ y \end{bmatrix}$.

Base on the support of theorem 1, the principle problem to be dealt with is to select the proper common positive definite matrix $P$ and the control force $K$. In this section, EBA is employed to search for the proper solutions. In this case, solutions obtained in the searching process can be classified as feasible and infeasible solutions.

It implies that the fitness function should better be designed in a binary operation form to answer to the need of this application. We design the fitness function according to the stability criterion derived from the LMI conditions via the Lyapunov function approach. The AND logical operation is used to produce the binary classification result on the obtained solutions. The fitness function is listed as follows:

$$F(P, K) = \begin{cases} 1, & \text{if } (A_1 - B_1 K)^TP + P(A_1 - B_1 K) > 0 \text{ and } P = P^T > 0. \\
0, & \text{otherwise} \end{cases}$$

The common positive definite matrix $P$ is always constrained to be symmetric when using EBA to change the elements inside it.

Moreover, a boundary condition is used at the initialization process for both matrices $P$ and $K$. The matrix $P$ is kept influencing by the same range of boundary conditions for producing feasible solutions in a suitable range, but not applied for the matrix $K$ because the total effect contributed to the system by the control force is relatively small. Parameters used in the experiment for EBA are listed in Table 1.

**CONCLUSION**

A simulation of the nonlinear inverted pendulum system is given at the last. The experimental result indicates that EBA with our proposed fitness function presents the 94.1% success rate in average for finding the feasible solutions.

**REFERENCES**


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Table 1 Parameters for EBA

<table>
<thead>
<tr>
<th>Boundary condition for matrix $P$ and $K$</th>
<th>[-5, 5]</th>
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<tr>
<td>Medium Material</td>
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<td>Number of Iteration</td>
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<td>Population size</td>
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