MODELING AND SIMULATION OF INVERTED PENDULUM SYSTEM USING MATLAB: OVERVIEW

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Abstract—Automatic Control is a growing field of study in the field of Mechanical Engineering. This covers the proportional, integral and derivative (PID) and state space control. The principal reasons for its popularity are its nonlinear and unstable characteristics. This report begins with an outline of the research into Inverted Pendulum system design and control along with mathematical modeling methods. This project presents the introduction and review of two dimensional inverted pendulum systems done in MATLAB environment.

Keywords—Inverted Pendulum, MATLAB,PID Controller, Simulink.

I. INTRODUCTION

The Inverted Pendulum is a classical control theory problem. It involves developing a system to balance an Inverted pendulum. For visualization purposes, this is similar to trying to balance a broomstick on a finger. There are three main subsystems that compose this design including the mechanical system, the feedback networkand a controller. The most fundamental case is when a pendulum is mounted on a cart which can move back and forth in one linear direction. The pendulum is then balanced in upright position by controlling the movement of the cart. The International Federation of Automatic Control (IFAC) Theory Committee in the year 1990 has determined a set of practical design problems that are helpful in comparing new and existing control methods and tools so that a meaningful comparison can be derived. The committee came up with a set of real world control problems that were included as "benchmark control problems". Out of which the cascade inverted pendula control problem is featured as highly unstable, and the toughness increases with increase in the number of links.The Inverted Pendulum is a single input multi output (SIMO) system with control voltage as input, cart position and pendulum angle as outputs.The Inverted Pendulum on a cart has been selected as the system on which linear state feedback controllers will be implemented.

1. It is the ideal study object of nonlinear system because it has the characteristics of inherent instability & inherent non-minimum phase characteristics.
2. The researches on Inverted Pendulum control System have an extensive application prospect in the robotics, attitude control in the satellite flight & general industrial applications.
3. As Inverted Pendulum is a natural nonlinear system so that complicated nonlinear controllers were applied to find the solution for its stabilization problem.

II. LITERATURE REVIEW

The inverted pendulum since is an important control problem which the researchers have been trying to solve worldwide for last few decades. Historically, the Inverted Pendulum was used first by seismologists in design of a "seismometer" in the year 1844 in Great Britain. Since, the system is inherently in unstable equilibrium when mounted on a stiff wire it can sense even the slightest of vibrations. In [1], stabilization of the cart pendulum system was carried out by linearization of the state model and designing a LQR after swing-up by energy based controller. The velocity states were less penalized compared to the position states in [2] so that the resulting stabilized system will have almost zero position as a zero velocity is only a secondary priority. This logic will lead to an almost upright system. In [3] two methods of controlling the Inverted pendulum parameter with only one control signal from motor. In one method, the cart's position is controlled beneath the pendulum's angle as shown in figure 1 and 2. And in another method; the pendulum's angle is controlled beneath the cart's position as shown in figure 4 and 5. By using PID controller and linear approximation the time response of the two methods algorithms under same condition was compared. The second method found to be better as oscillations and overshoot is less, and the convergence is faster. By moving a car along horizontal track, balancing of Inverted pendulum was done by Hong Huo. Mathematical model of single Inverted pendulum was built and experiment was established by using controller. A non-linear controller is described in [4], in which the controller swings up the pendulum from the pendant position and stabilizes the pendulum in the unstable equilibrium and simultaneously restricts the cart excursion on the track. A simple controller for balancing the inverted pendulum to the upper equilibrium point and minimize the cart position to zero is discussed in [5]. The inverted pendulum (IP) is highly motivated by applications such as the control of rockets and the antiseismic control of buildings. The goal of controlling IP is to balance the pendulum in upright position when it initially starts with some nonzero angle off the vertical position. This is a very typical nonlinear control problem, and many techniques already exist for its solution [6]. The behavior of an Inverted Pendulum structures during earthquakes are discussed in [7], in which a real practical example of Chilean earthquake is presented. During May 1960, elevated water tanks have not much affected in comparison to more stable appearing small structures as described in a companion paper by W.K. Cloud and K.V. Stienbrugge respectively. The overturning of rocking block against a constant horizontal acceleration, by a single sine pulse (earthquake type excitation) and against a constant horizontal force are analyzed and shown that stability of elevated structure is more against constant horizontal acceleration and less against constant horizontal force in comparison with more stable small structures. Inverted pendulum is defined and separated in two axis subsystem and control the angle with controllers and optimum parameters are obtained in [8]. Only angle is controlled but angle and/or position can be controlled.

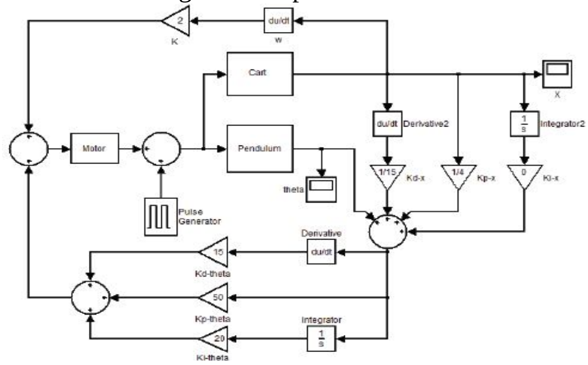

Fig: - 1 MATLAB Simulink Program in order to control the cart's position beneath the pendulum's angle.

Fig:-2Time Response of Inverted Pendulum system while cart position is controlled

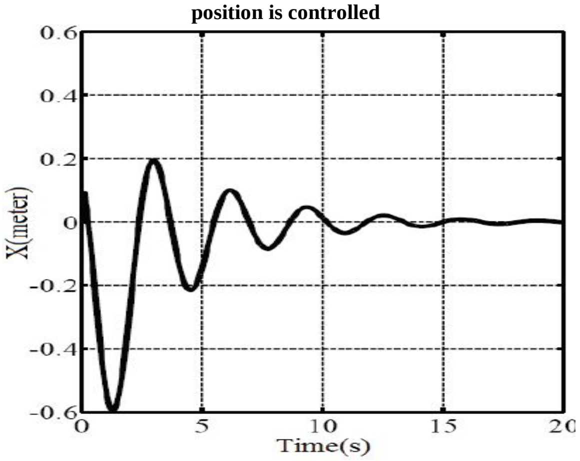

Fig: - 2(a) Time Response of pendulum's angle

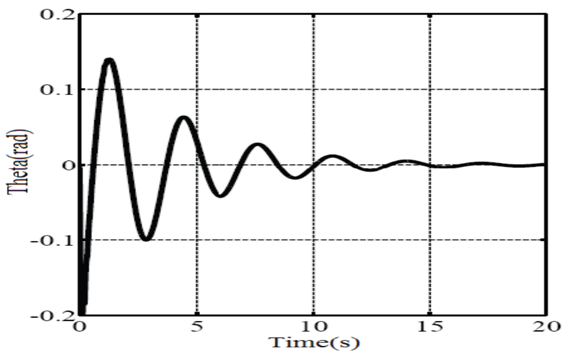

Fig:-2(b) Time Response of cart's position

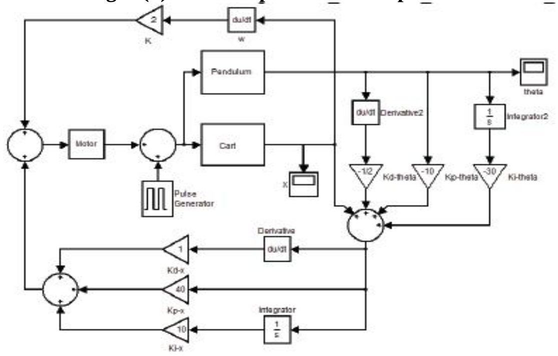

Fig: - 3 MATLAB Simulink Program in order to control the pendulum's angle beneath the cart's position.

Fig: - 4 Time Response of Inverted Pendulum system while pendulum's angle is controlled

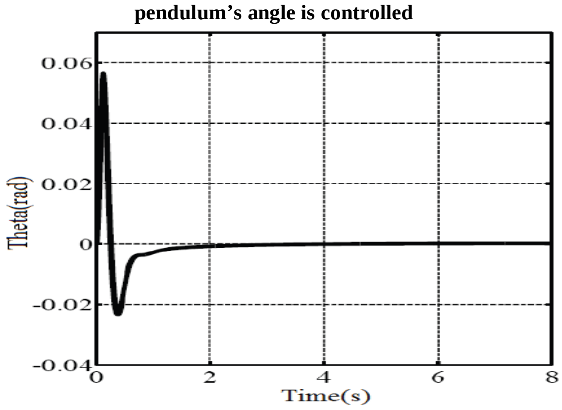

Fig: - 4(a) Time Response of pendulum's angle

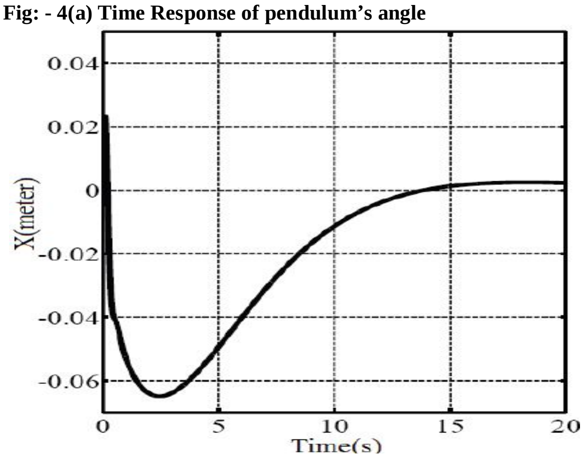

Fig: - 4(b) Time Response of pendulum's angle

In [9] the reflection of Inverted Pendulum benchmark was given. A survey of different control approaches was given and provides the overall scenario of developments for stabilizing the Inverted Pendulum. The model has defined the ideas in control theory. As we know inverted pendulum is an inherently unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control theory is required. The Inverted Pendulum is an invaluable tool for the effective evaluation and comparison various control theories.

III. DYNAMIC MODELLING OF CART SYSTEM

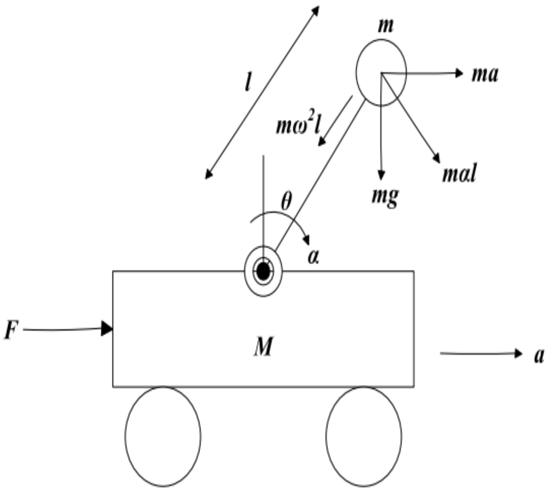

Fig: - 5 Cart-Stick System With Relevant Forces And Accelerations

The mathematical model of cart-stick problem is derived by using the Newtonian and lagrangian approach. It can be verified that equation coming from both approach are exactly same and hence warrants the validity of derived mathematical model.

IV. NEWTONIAN APPROACH

Using Newton's second law of motion to derive the equation of motion. Thus applying $\Sigma F = Ma$ on the stick and cart separately.

$F + N = Ma$And

$N = Mw^2 l + \sin\theta - ma - m\alpha l\cos\theta$

The horizontal reaction force between the stick and cart N needs to be eliminated and thus

$F = (m + M)a - mw^2 l\cos\theta + m\alpha l\cos\theta$.....(1)

Now the total external torque on the mass m about the point of contact between the cart and stick is $mgl\sin\theta$ and the moment of inertia about the same point is ml^2. Hence applying $\Sigma T = I\alpha$ about the point of contact, we find

$mgl\sin\theta = ml^2(\alpha + a\cos\theta)$

Or,$mg\sin\theta = m\alpha l + ma\cos\theta$

Or, $g\sin\theta - \alpha l - \cos\theta = 0$ .....(2)

Equation (1) and (2) are sufficient to describe the dynamics of the cart-stick system. Hence N has been eliminated.

The lagrangian approach allows one to deal with scalar energy functions rather than vector forces and accelerations as in Newtonian approach, thus reducing the chances of error.The first step in deriving the equations of motion using the lagrangian approach is to derive the, £.

$£ = T - V$

Where, T = Total K.E. of the system

V = Total P.E. of the system

The X and Y co-ordinates of m at any given time would be $(x + l\sin\theta)$ and $(l\cos\theta)$ respectively. The X and Y components of velocity of m can be obtained by differentiating the co-ordinates of m.

Therefore, $v_x = (\quad + l\dot{\theta}\cos\theta)$ and $v_y = -l\dot{\theta}\sin\theta$. Hence the total K.E. of the system, T can be written as

$$T = \frac{1}{2}M\quad^2 + \frac{1}{2}m[(\quad + l\theta\cos\theta)^2 + \left(-l\dot{\theta}\sin\theta\right)^2]$$ .....(3)

Considering the cart to be at ground reference level and m to be the only mass above this reference level, the total P.E. of the system, V can be written as

$V = mgl\cos\theta$.....(4)

Equation (3) and (4) can be used to find lagrangian, £.

$£ = \frac{1}{2}(m + M)\quad^2 + ml\quad\cos\theta + \frac{1}{2}ml^2\theta^2 - mgl\cos\theta$.....(5)

As we know the cart-stick system has two degree of freedom and F is the only force applied to the system that is capable of changing x or $\dot{x}$. However no external force is applied to the system affects θ directly.

Hence, the equations of motion for the cart-stick system will be derived by simplifying

$\frac{d}{dt}\frac{d£}{d\dot{x}} - \frac{d£}{dx} = F$And

$$\frac{d}{dt}\frac{d£}{d\dot{\theta}} - \frac{d£}{d\theta} = 0$$

The first equation of motion can be found by simplifying

$\frac{d}{dt}\frac{d£}{d\dot{x}} - \frac{d£}{dx} = F$.....(6)

Or, $\frac{d}{dt}\left[(M + m)\quad + ml\dot{\theta}\cos\theta\right] = F$

Or, $(M + m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml^2\sin\theta = F$

Or,$F = (M + m)a - mw^2 l\sin\theta + m\alpha l\cos\theta$.....(7)

The second equation of motion can be found by simplifying

$\frac{d}{dt}\frac{d£}{d\theta} - \frac{d£}{d\theta} = 0$.....(8)

Or,

d/dt [ml cosθ + ml2θ] – [ml θ sinθ + mglsinθ] = 0

Or, $ml(\ddot{x}\cos\theta - \theta\sin\theta + l\ddot{\theta} + \theta\sin\theta) = 0$

Or $g\sin\theta - \alpha l - ga\cos\theta = 0$ …..(9)

It can be seen that (1) and (2) are exactly the same as (7) and (9) respectively. This verification signifies that the dynamic modeling of cart-stick system has been done correctly and that these equations are sufficient to describe the dynamics of the cart-stick system.

CONCLUSION

From the above review we come to know the different aspect, research and development of inverted pendulum. We also know the application and advantages of studying and modeling Inverted Pendulum. [10] Showed that any linear feedback which can control the linear approximation of model to an asymptotic stability also can control the originally nonlinear system to asymptotic stability. So,it provided theoretical basis for the linear controller design of nonlinear inverted pendulum system thereby simplifying the usual complexities of the controller design for nonlinear systems. Hence, this method can be extended to controller design of other nonlinear systems too.

REFERENCES

[1] D. Chatterjee, A. Patra, H. K. Joglekar, "Swing-up and Stabilization of a cart-pendulum system under restricted cart length", Systems and Control Letters, vol. 47, pp. 355-364, July 2002.

[2] S. A. Campbell, S. Crawford, K. Morris, "Friction and the Inverted Pendulum Stabilization Problem", J. of Dynamic System Measurement and Control, vol.130, no.5, 054502 (7 pages), ASME, August 2008.

[3] Kaveh Razzaghi, "A new approach on stabilization Control of an inverted pendulum, Using PID controller." 2011 International Conference on Control Robotics and Cybernetics (ICCRC 2011).

[4] Q. Wei, W.P. Dayawansa, W.S. Levine, "Nonlinear controller for an inverted pendulum having restricted travel," Automatica, vol. 31, no. 6, pp. 841-850, June 1995.

[5] R. Lozano, I. Fantoni, D. J. Block, "Stabilization of the inverted pendulum around its homoclinic orbit," Systems & Control Letters, vol. 40, no. 3, pp. 197-204, July 2000.

[6] Maravall, D., Zhou, C. and Alonso, J. (2005), Hybrid fuzzy control of the inverted pendulum via vertical forces. Int. J. Intell. Syst., 20: 195–211.

[7] George W. Housner "The Behavior of Inverted Pendulum structures during earthquakes," Bulletin of the seismological society of America. Vol. 53, No 2, pp.403-417. February, 1963.

[8] M. Adra and H. Kuşç u, "Modeling of two -dimensional Inverted Pendulum in MATLAB/Simulink"

[9] Ofla Boubalker, "The Inverted Pendulum: A fundamental Benchmark in Control Theory and Robotics," 2012 International Conference on Education and e-Learning Innovations.

[10] W.Zhang and Hong-qi-Wang. Research on Key problems in the controller design for inverted pendulum system. Fourth International Conference on Intelligent computation technology and Automation. IEEE. DOI 10.11.09/ICICTA. 2011.135

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