THE WONDERFULS OF FIBONACCI NUMBERS IN THE HIDDEN NATURE

MARZIEH GHAEDI

Dept. of Hormozgan University, Bandarabbas, Iran

Abstract: In this paper a certain amount depending on the proximity to the Fibonacci sequence, the golden ratio handle Darymma this particular call and the relationship that we have found with nature. The purpose of this article is wonderful examples of practical nature are mathematical Rules.
This has led to some amazing properties, Fibonacci numbers to know the hidden nature of carrier codes.

I. INTROUDUCTION

In the West, the Fibonacci sequence first appears in the book *Liber Abaci* (1202) by Leonardo of Pisa, known as Fibonacci.[4] Fibonacci considers the growth of an idealized (biologically unrealistic) rabbit population, assuming that: a newly born pair of rabbits, one male, one female, are put in a field; rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits; rabbits never die and a mating pair always produces one new pair (one male, one female) every month from the second month on. The puzzle that Fibonacci posed was: how many pairs will there be in one year?

- At the end of the first month, they mate, but there is still only 1 pair.
- At the end of the second month the female produces a new pair, so now there are 2 pairs of rabbits in the field.
- At the end of the third month, the original female produces a second pair, making 3 pairs in all in the field.
- At the end of the fourth month, the original female has produced yet another new pair, the female born two months ago produces her first pair also, making 5 pairs.

At the end of the *n*th month, the number of pairs of rabbits is equal to the number of new pairs (which is the number of pairs in month $n-2$) plus the number of pairs alive last month ($n-1$). This is the *n*th Fibonacci number.[14]
The name "Fibonacci sequence" was first used by the 19th-century number theorist Édouard Lucas.[15]

The **Fibonacci numbers** or **Fibonacci series** or **Fibonacci sequence** are the numbers in the following integer sequence:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 144,\ \ldots$$

By definition, the first two numbers in the Fibonacci sequence are 0 and 1, and each subsequent number is the sum of the previous two.
In mathematical terms, the sequence F_n of Fibonacci numbers is defined by the recurrence relation

$$F_n = F_{n-1} + F_{n-2},$$

with seed values[3]

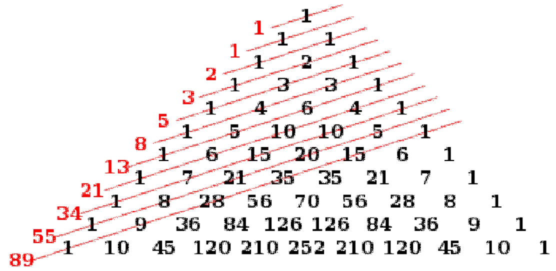
$$F_0 = 0,\ F_1 = 1.$$

The Fibonacci sequence is named after Leonardo Fibonacci.
Fibonacci numbers are closely related to Lucas numbers in that they are a complementary pair of Lucas sequences. They are intimately connected with the golden ratio; for example, the closest rational approximations to the ratio are 2/1, 3/2, 5/3, 8/5, ... Applications include computer algorithms such as the Fibonacci search technique and the Fibonacci heap data structure, and graphs called Fibonacci cubes used for interconnecting parallel and distributed systems. They also appear in biological settings,[8] such as branching in trees, phyllotaxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple,[9] the flowering of artichoke, an uncurling fern and the arrangement of a pine cone.[10]
The Fibonacci sequence appears in Indian mathematics, in connection with Sanskrit prosody.[6][11] In the Sanskrit oral tradition, there was much emphasis on how long (L) syllables mix with the short (S), and counting the different patterns of L and S within a given fixed length results in the Fibonacci numbers; the number of patterns that are *m* short syllables long is the Fibonacci number F_{m+1}.[7]
The sequence had been described earlier. By modern convention, the sequence begins either with $F_0 = 0$ or with $F_1 = 1$. The *Liber Abaci* began the sequence with $F_1 = 1$, without an initial 0.

Susantha Goonatilake writes that the development of the Fibonacci sequence "is attributed in part to Pingala (200 BC), later being associated with Virahanka (c. 700 AD), Gopāla (c. 1135), and Hemachandra (c. 1150)".[5] Parmanand Singh cites Pingala's cryptic formula *misrau cha* ("the two are mixed") and cites scholars who interpret it in context as saying that the cases for *m* beats (F_{m+1}) is obtained by adding a [S] to F_m cases and [L] to the F_{m-1} cases. He dates Pingala before.

II. OCCURRENCES IN MATHEMATICS

The Fibonacci numbers are the sums of the "shallow" diagonals (shown in red) of Pascal's triangle.



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The Fibonacci numbers occur in the sums of "shallow" diagonals in Pascal's triangle.[18]

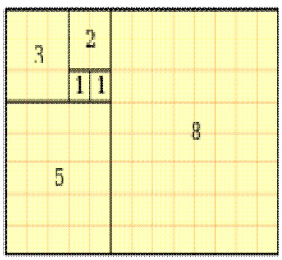
$$F_n = \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n-k-1}{k}.$$

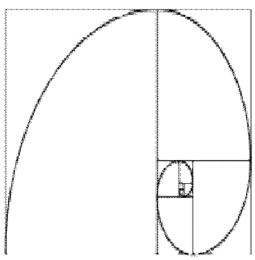
The Fibonacci numbers can be found in different ways in the sequence of binary strings.

- The number of binary strings of length n without consecutive 1s is the Fibonacci number F_{n+2}. For example, out of the 16 binary strings of length 4, there are $F_6 = 8$ without consecutive 1s – they are 0000, 0100, 0010, 0001, 0101, 1000, 1010 and 1001. By symmetry, the number of strings of length n without consecutive 0s is also F_{n+2}.
- The number of binary strings of length n without an odd number of consecutive 1s is the Fibonacci number F_{n+1}. For example, out of the 16 binary strings of length 4, there are $F_5 = 5$ without an odd number of consecutive 1s – they are 0000, 0011, 0110, 1100, 1111.
- The number of binary strings of length n without an even number of consecutive 0s or 1s is $2F_n$. For example, out of the 16 binary strings of length 4, there are $2F_4 = 6$ without an even number of consecutive 0s or 1s – they are 0001, 1000, 1110, 0111, 0101, 1010.

III. FIGER OF FIBONACCI NUMBER

A tiling with squares whose side lengths are successive Fibonacci numbers in the fact





An approximation of the golden spiral created by drawing circular arcs connecting the opposite corners of squares in the Fibonacci tiling; this one uses squares of sizes 1, 1, 2, 3, 5, 8, 13, 21, and 34.

Relation to the golden ratio

Like every sequence defined by a linear recurrence with constant coefficients, the Fibonacci numbers have a closed-form solution. It has become known as Binet's formula, even though it was already known by Abraham de Moivre

$$F_n = \frac{\varphi^n - \psi^n}{\varphi - \psi} = \frac{\varphi^n - \psi^n}{\sqrt{5}}$$

where

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.61803\,39887\cdots$$

is the golden ratio (sequence A001622 in OEIS), and

$$\psi = \frac{1-\sqrt{5}}{2} = 1-\varphi = -\frac{1}{\varphi} \approx -0.6180339887\cdots$$

[20]

Since $\psi = -\frac{1}{\varphi}$, this formula can also be written as

$$F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$$

To see this,[21] note that φ and ψ are both solutions of the equations

$x^2 = x + 1,\ x^n = x^{n-1} + x^{n-2},$

so the powers of φ and ψ satisfy the Fibonacci recursion. In other words

$$\varphi^n = \varphi^{n-1} + \varphi^{n-2}$$

and

$\psi^n = \psi^{n-1} + \psi^{n-2}$.

It follows that for any values a and b, the sequence defined by

$$U_n = a\varphi^n + b\psi^n$$

satisfies the same recurrence

$U_n = a\varphi^{n-1} + b\psi^{n-1} + a\varphi^{n-2} + b\psi^{n-2} = U_{n-1} + U_{n-2}.$

If a and b are chosen so that $U_0 = 0$ and $U_1 = 1$ then the resulting sequence U_n must be the Fibonacci sequence. This is the same as requiring a and b satisfy the system of equations:

$$\begin{cases} a + b = 0 \\ \varphi a + \psi b = 1 \end{cases}$$

which has solution

$$a = \frac{1}{\varphi - \psi} = \frac{1}{\sqrt{5}},\ b = -a$$

producing the required formula.

Limit of consecutive quotients[edit source | editbeta]

Johannes Kepler observed that the ratio of consecutive Fibonacci numbers converges. He wrote that "as 5 is to 8 so is 8 to 13, practically, and as 8 is to 13, so is 13 to 21 almost", and concluded that the limit approaches the **Computation by rounding[edit source | editbeta]**

Since

$$\frac{|\psi|^n}{\sqrt{5}} < \frac{1}{2}$$

for all $n \geq 0$, the number F_n is the closest integer to

$$\frac{\varphi^n}{\sqrt{5}}.$$

Therefore it can be found by rounding, or in terms of the floor function:

$$F_n = \left\lfloor \frac{\varphi^n}{\sqrt{5}} + \frac{1}{2} \right\rfloor,\ n \geq 0.$$

Or the nearest integer function:

$$F_n = \left[\frac{\varphi^n}{\sqrt{5}} \right],\ n \geq 0.$$

Similarly, if we already know that the number $F > 1$ is a Fibonacci number, we can determine its index within the sequence by

$$n(F) = \left\lfloor \log_\varphi \left(F \cdot \sqrt{5} + \frac{1}{2} \right) \right\rfloor$$

golden ratio φ.[22][23]

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$$

This convergence does not depend on the starting values chosen, excluding 0, 0. For example, the initial values 19 and 31 generate the sequence 19, 31, 50, 81, 131, 212, 343, 555 ... etc. The ratio of consecutive terms in this sequence shows the same convergence towards the golden ratio.

In fact this holds for any sequence that satisfies the Fibonacci recurrence other than a sequence of 0's. This can be derived from Binet's formula.

Another consequence is that the limit of the ratio of two Fibonacci numbers offset by a particular finite deviation in index corresponds to the golden ratio raised by that deviation.

IV. RECOGNIZING FIBONACCI NUMBERS

The question may arise whether a positive integer x is a Fibonacci number. This is true if and only if one or both of $5x^2 + 4$ or $5x^2 - 4$ is a perfect square This is because Binet's formula above can be rearranged to give

$$n = \log_\varphi \left(\frac{F_n\sqrt{5} + \sqrt{5F_n^2 \pm 4}}{2} \right)$$

(allowing one to find the position in the sequence of a given Fibonacci number)

This formula must return an integer for all n, so the expression under the radical must be an integer (otherwise the logarithm does not even return a rational number).

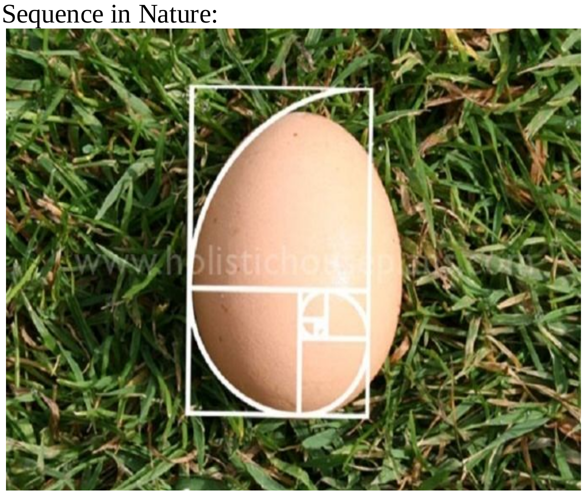
V. COMBINATORIAL IDENTITIES

Most identities involving Fibonacci numbers can be proven using combinatorial arguments using the fact that F_n can be interpreted as the number of sequences of 1s and 2s that sum to $n - 1$. This can be taken as the definition of F_n, with the convention that $F_0 = 0$, meaning no sum adds up to −1, and that $F_1 = 1$, meaning the empty sum "adds up" to 0. Here, the order of the summand matters. For example, 1 + 2 and 2 + 1 are considered two different sums.

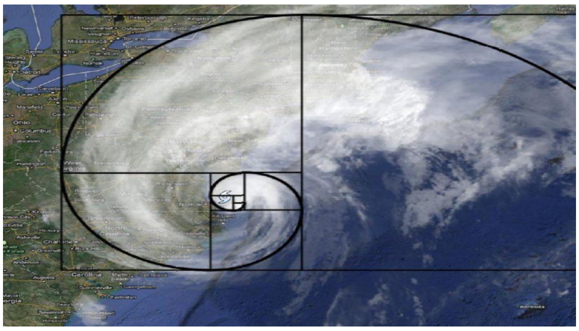
For example, the recurrence relation

$$F_n = F_{n-1} + F_{n-2},$$

or in words, the nth Fibonacci number is the sum of the previous two Fibonacci numbers, may be shown by dividing the $F(n)$ sums of 1s and 2s that add to $n-1$ into two non-overlapping groups. One group contains those sums whose first term is 1 and the other those sums whose first term is 2. In the first group the remaining terms add to $n - 2$, so it has $F(n-1)$ sums, and in the second group the remaining terms add to $n-3$, so there are $F(n-2)$ sums. So there are a total of $F(n-1) + F\ n-2)$ sums altogether, showing this is equal to $F\ (n)$. The Fibonacci Sequence in Nature:

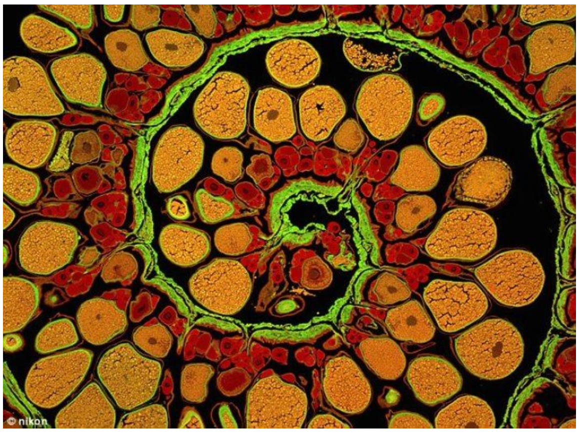


The fibonacci spiral appears not only in the perfect nautilus shell,

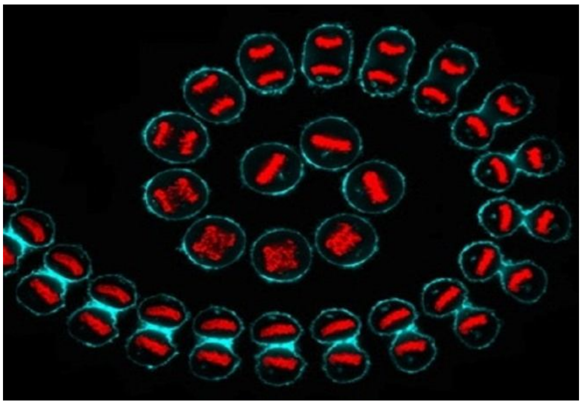


but in events and objects viewed from a far. An energy system in the shape of a fibonacci moves with limited losses. Hurricane Irene. imgur.com

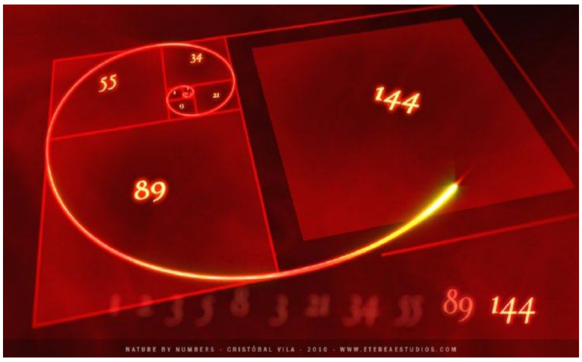
The fibonacci as some of the largest structures in the universe. Spiral galaxies are the most common galaxy shape. Galaxies group together in superclusters and superclusters group together in walls. Currently the largest known structures are these walls or filaments of numerous superclusters that are gravitationally bound and separated by large areas of void. The Milky Way's dust obstructs us from seeing the depth of these filaments or sheets, so we do not yet know the exact shape of these walls. www.spacetelescope.org



The fibonacci appears in the smallest, to the largest objects in nature. It is a way for information to flow in a very efficient manner. Here, a microscopic view of the ovary of an Anglerfish. Nikon's It's a Small World Competition. www.dailymail.co.uk



Cancer cell division. This composite confocal micrograph uses time-lapse microscopy to show a cancer cell (HeLa) undergoing cell division (mitosis). The DNA is shown in red, and the cell membrane is shown in cyan. The round cell in the centre has a diameter of 20 microns. Credit Kuan-Chung Su, LRI, www.wellcomeimageawards.org



The mathematics of the golden ratio and of the Fibonacci sequence are intimately interconnected. The Fibonacci sequence is a recursive sequence, generated by adding the two previous numbers in the sequence.: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987...

If you were to draw a line starting in the right bottom corner of a golden rectangle within the first square, and then touch each succeeding multiple square's outside corners, you would create a fibonacci spiral.

Fibonacci as starting point of life. Image: www.holistichouseplans.com

Romanesque brocolli is a striking example of the fibonacci. www.flickr.com

Spiral aloe. Numerous cactus display the fobonacci spiral. www.flickr.com

Sunflower. www.thestrong.org

Marlborough Rock Daisy by Sid Mosdell. www.flickr.com

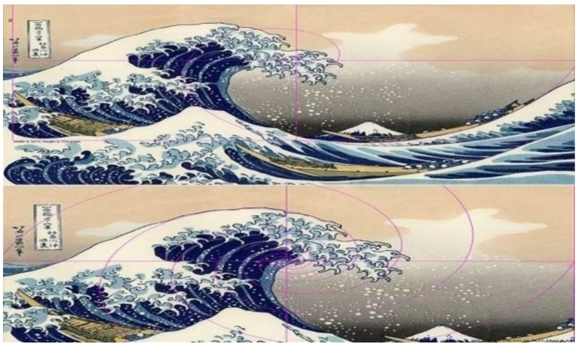
All pinecones display a fibonacci sequence.

A monarch caterpillar about to form a chrysalis. natureremains.blogspot.com

Fibonacci and armor = very safe. www.fieldherpforum.com

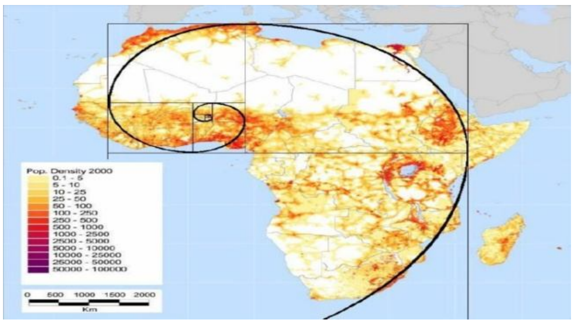
Fibonacci in spores. A fiddlehead or koru. Photo by Sid Mosdell. www.flickr.com

Snails and fingerprints. Images: artcatalyst.blogspot.com, www.123rf.com



Fibonacci in the wave. artcatalyst.blogspot.com

Water falls into the shapes of a fibonacci during numerous events. Another example would be a vortex. fibonacci-seri.es



One blogger has applied the Fibonacci sequence to population density and land mass. In Africa the majority of highly populated cities fall on or close to where the spiral predicts.
earelephant.blogspot.com

Shell Fossil via: www.123rf.com

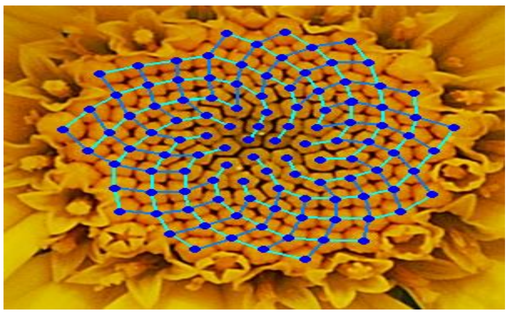
VI. THE BEE ANCESTRY CODE

Fibonacci numbers also appear in the description of the reproduction of a population of idealized honeybees, according to the following rules:

- If an egg is laid by an unmated female, it hatches a male or drone bee.
- If, however, an egg was fertilized by a male, it hatches a female.

Thus, a male bee always has one parent, and a female bee has two. If one traces the ancestry of any male bee (1 bee), he has 1 parent (1 bee), 2 grandparents, 3 great-grandparents, 5 great-great-grandparents, and so on. This sequence of numbers of parents is the Fibonacci sequence. The number of ancestors at each level, F_n, is the number of female ancestors, which is F_{n-1}, plus the number of male ancestors, which is F_{n-2}. This is under the unrealistic assumption that the ancestors at each level are otherwise unrelated

Further information: Patterns in nature and Phyllotaxis



Yellow Chamomile head showing the arrangement in 21 (blue) and 13 (aqua) spirals. Such arrangements involving consecutive Fibonacci numbers appear in a wide variety of plants.

Fibonacci sequences appear in biological settings,[8] in two consecutive Fibonacci numbers, such as branching in trees, arrangement of leaves on a stem, the fruitlets of a pineapple,[9] the flowering of artichoke, an uncurling fern and the arrangement of a pine cone.[10] In addition, numerous poorly substantiated claims of Fibonacci numbers or golden sections in nature are found in popular sources, e.g., relating to the breeding of rabbits in Fibonacci's own unrealistic example, the seeds on a sunflower, the spirals of shells, and the curve of waves.[51] The Fibonacci numbers are also found in the family tree of honeybees.[52]

The DNA molecule measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix spiral. 34 and 21, of course, are numbers in the Fibonacci series and their ratio, 1.6190476, closely approximates 1.6180339.

Przemysław Prusinkiewicz advanced the idea that real instances can in part be understood as the expression of certain algebraic constraints on free groups, specifically as certain Lindenmayer grammars.

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2. Singh, Parmanand (1985), "The So-called Fibonacci numbers in ancient and medieval India", *Historia Mathematica* **12** (3): 229–44, doi:10.1016/0315-0860(85)90021-7
3. Knuth, Donald (2006), *The Art of Computer Programming*, 4. Generating All Trees – History of Combinatorial Generation, Addison–Wesley, p. 50, ISBN 978-0-321-33570-8, "it was natural to consider the set of all sequences of [L] and [S] that have exactly m beats. ...there are exactly Fm+1 of them. For example the 21 sequences when *m* = 7 are: [gives list]. In this way Indian prosodists were led to discover the Fibonacci sequence, as we have observed in Section 1.2.8 (from v.1)".
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5. Jones, Judy; Wilson, William (2006), "Science", *An Incomplete Education*, Ballantine Books, p. 544, ISBN 978-0-7394-7582-9
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7. Knuth, Donald (1968), *The Art of Computer Programming* **1**, Addison Wesley, ISBN 81-7758-754-4, "Before Fibonacci wrote his work, the sequence Fn had already been discussed by Indian scholars, who had long been interested in rhythmic patterns... both Gopala (before 1135□AD) and Hemachandra (c.□1150) mentioned the numbers 1,2,3,5,8,13,21 explicitly [see P. Singh Historia Math 12 (1985) 229–44]" p. 100 (3d ed)..."
8. Agrawala, VS (1969), Pā□inikālīna Bhāratavar□a *(Hn.). Varanasi-I: TheChowkhamba Vidyabhawan*, "SadgurushiShya writes that Pingala was a younger brother of Pā□ini [Agrawala 1969, lb]. There is an alternative opinion that he was a maternal uncle of Pā□ini [Vinayasagar 1965, Preface, 121. ... Agrawala [1969, 463–76], after a careful investigation, in which he considered the views of earlier scholars, has concluded that Pā□ini lived between 480 and 410 BC"
9. Knott, Ron. "Fibonacci's Rabbits". University of Surrey Faculty of Engineering and Physical Sciences.
10. Gardner, Martin (1996), *Mathematical Circus*, The Mathematical Association of America, p. 153, ISBN 0-88385-506-2, "It is ironic that Leonardo, who made valuable contributions to mathematics, is remembered today mainly because a 19th-century French number theorist, Édouard Lucas... attached the name Fibonacci to a number sequence that appears in a trivial problem in Liber abaci".
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