FREQUENCY ANALYSIS OF CARBON NANOTUBE BASED MASS SENSOR

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Abstract— This paper presents finite element modeling (FEM) of carbon nanotube (CNT) and explores the suitability of CNT based nanoresonator to be used as mass sensor. Simulation of CNT based mass sensor has been carried out for two different configurations namely, cantilevered and bridged boundary conditions using FEM based software ABAQUS.The effect of varying length of CNT as well as mass on resonance frequency have been simulated and observed that shorter length of CNT is more sensitive for sensing mass as small as of the order of zeptogram level. Also the influence of position of attached mass on resonance frequency has been explored in the present work. Based on analysis, it has been observed that mass up to 10E-25kg or even less can be detected using CNT based nanoresonator.

Index Terms— Carbon Nanotube, Mass Sensor, Euler Bernoulli Beam, Finite Element Method.

I. INTRODUCTION

In 1991, discovery of the carbon nanotubes has attracted many researches, SumioIijima[1] observed Single walled Carbon nanotubes (SWCNTs). Carbon nanotubes (CNTs) areconsidered to be a cylindrical hollow tubular structure at molecular level.CNT is made by rolling up of grapheme sheet. Carbon nanotube is 100 times stronger than steel and very flexible. Carbon Nanotubes have many structures, different in length and diameter. Although they are made from essentially the same graphite sheet, their electrical properties differ depending on these variations, acting either as metals or as semiconductors [2]

Literature review shown an increased utilization of modelling methods based on elastic continuum mechanics theories for studying the vibration of carbon nanotube.The extraordinary properties of carbon nanotubes (CNTs) give them potential in various applications and open an improbable range of all kind of applications.R.Saitoet et al. [3]has published introductory textbook for researchers from various fields of science who wish to learn about carbon nanotubes. The researcher is pointing on the basic principles behind the physical properties of carbon nanotube.J. Han et al.[4]have taken a comprehensive look at this diverse and dynamic subject, Carbon Nanotubes: Engineering and Applications describes the field's various aspects, including characteristics, growth and processing techniques. A.P. Moravsky et al. [5] discuss that SWNTs are generally consisting of a few dozen carbon nanotubes in form of bundles which is tightly compounded in a honeycomb lattice with an average separation between tube axes of approximately 1.5nm.C.Kittelet al.[6] discuss that by exploring the electrical properties of carbon nanotubes, it is possible to explore the noticeable distinctions regarding conductivity between metals and semiconductors presented in carbon nanotube structures.R. Chowdhuryet al. [7] discuss that the potential of SWNTs as a mass sensor is explored. CNTs are modelled by using continuum based approach forboth cantilevered and bridged CNTs in the present paper. The suitability of same to be used as mass sensor has been explored.

II. MODLING OF CARBON NANOTUBE

The objective is to develop the sensor based on SWNTs so as to be able to identify the mass that may be attached at the centre for fixed-fixed nanotube and free end for fixed-free nanotube. In order to perform the vibration analysis of the system SWNTs and mass, the added mass can be analysed as a spherical mass when its mass is much less than the mass of the SWCNT. Using various approximations for vibrating cantilever beams and bridge beams with spherical masses at their tip various researchers have presented in [8-15].

In the present work, the influence of different boundary conditions on resonance frequency has been explored.

(a) **Cantilever Boundary Condition:-** For cantilever configuration as shown in fig.1 mass frequency relationship has been derived. The equivalent stiffness and equivalent mass for cantilever configuration are as follows:

$$K_{CNT}=\frac{3E_{CNT}I_{CNT}}{L_{CNT}^{3}} \quad (1)$$

$$M_{CNT}=\frac{33}{140}\rho AL+M_{ATTECHED}(2)$$

Using Eqs. (1) and (2) resonance frequency of CNT based mass sensor has been obtained as:

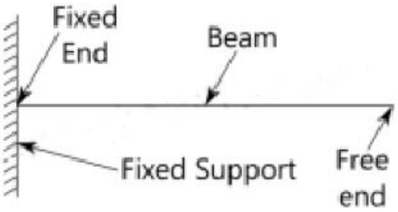


Figure 1: Cantilever CNT beam

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\frac{3EI}{L^3}}{\frac{33}{140}\rho AL+M}} \quad (3)$$

$$f_n = \frac{1}{2\pi}\frac{\alpha^2\beta}{\sqrt{1+\Delta M}} (4)$$

Where; $\alpha^2 = \sqrt{\frac{140}{11}}$ =1.888, $\beta = \sqrt{\frac{EI}{\rho AL^4}}$ and $\Delta M = \frac{M}{\rho AL}\mu$all are constants, in which,$\mu = \frac{140}{33}$ for cantilever beam. *E*, *I*, ρ, *A* and *L* represent Young's modulus, moment of inertia, density, cross sectional area and length of CNT respectively whereas *M* is the attached mass on CNT.

Similarly for no mass attached on CNT the resonance frequency obtained by simply putting M=0 in Eq. (4) as:

$$f_n = \frac{1}{2\pi}\alpha^2\beta \quad (5)$$

(b) **Bridge Boundary Condition:-**Similar to cantilever configuration, mass frequency relationship for bridged configuration has also been derived as dipicted in fig.2. The equivalent stiffness and equivalent mass for cantilever configuration are as follows:

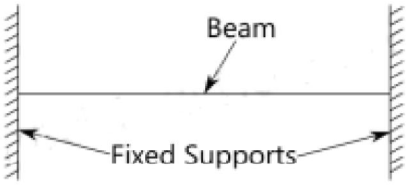


Figure 2: Bridge CNT beam

$$K_{CNT} = \frac{192E_{CNT}I_{CNT}}{L_{CNT}^3} \quad (6)$$

$$M_{CNT} = \frac{13}{35}\rho AL+M_{ATTECHED} \quad (7)$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\frac{192EI}{L^3}}{\frac{13}{35}\rho AL+M}} = \frac{1}{2\pi}\frac{\alpha^2\beta}{\sqrt{1+\Delta M}} (8)$$

Where;

$\alpha^2 = \sqrt{\frac{6720}{13}}$ =4.888, $\beta = \sqrt{\frac{EI}{\rho AL^4}}$, and$\Delta M = \frac{M}{\rho AL}\mu$, in which $\mu = \frac{35}{13}$ for bridged configuration.

III. FINITE ELEMENT MODELING APPROACH

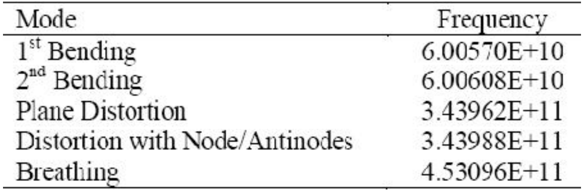
The FEM has become a powerful numerical method for analysing physical phenomena in the fields of structural, solid and fluid mechanics. In the last almost four decades, this methodology has become the prevalent technique used for analysing physical phenomena in the field of model structural, solid and fluid mechanics as well as for solution of field structural problems. The FEM is a useful tool because one can use it to find out facts or study the process in a way not possible with any other tool.

Further, the computational approach can be considered as an important tool in the development of nano composites and identifying their characteristics. It helps to understand and design these unique materials. By means of finite element methodology, it is possible to identify thermal, mechanical and electrical properties and to study structural responses of nano materials under various conditions.

IV. RESULT AND DISCUSSION

In order to investigate the effect of varying boundary conditions aCNT of length 10 nm has been analysed in FEM based software ABAQUS 12.0. A mass of 0.1 zeptogram is applied at free end of cantilevered and at the centre of bridged nano resonator in order to investigate the resonance behaviour of same to be used as mass sensor. Various modes of vibrations with varying frequencies have been observed and entered in Table .1.These patterns of vibrations are obtained for length of 10 nm and attached mass at free end as 0.1 zg as shown in Figure .3.

Table .1 Resonance frequencies of first five vibration modes for cantilever CNT of length 10nm with mass 0.1 zg at free end



Mode	Frequency
1^{st} Bending	6.00570E+10
2^{nd} Bending	6.00608E+10
Plane Distortion	3.43962E+11
Distortion with Node/Antinodes	3.43988E+11
Breathing	4.53096E+11

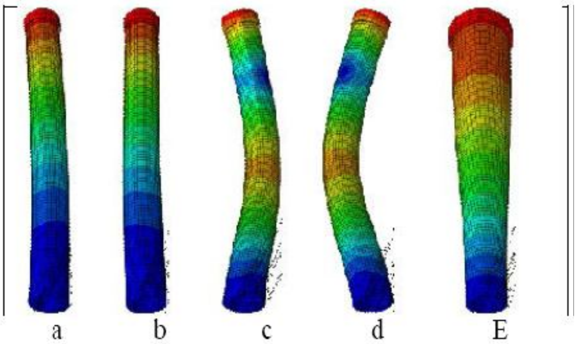


Figure 3: Mode shapes for cantilever CNT with mass 0.1 zg

Similarly simulation of 10 nm long CNT for bridged configuration with additional mass (0.1 zg) attached at centre has been carried out in order to investigate the resonance behaviour of same. The first five vibration patterns as shown in Figure .4 and their corresponding frequency values have entered in Table .2.

Table .2 Resonance frequencies of first five vibration modes for bridged CNT of length 10nm with mass 0.1 zg at centre

Mode	Frequency
1st Bending	3.41048E+11
2nd Bending	3.41569E+11
Plane distortion	8.31054E+11
Distortion withNode/Antinodes	8.31255E+11
Breathing	9.06258E+11

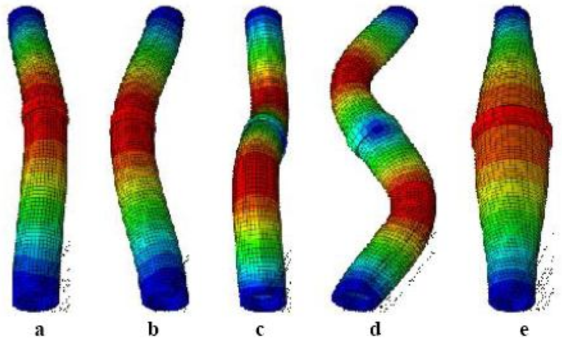


Figure 4: Mode shapes for cantilever CNT with mass 0.1 zg

Simulation results revels that resonance frequency for bridged configuration is higher suggesting its more suitability as mass censor as compared to cantilevered configuration.

Further, simulation has been carried out to investigate the effect of varying mass on resonance frequency for both configurations and observed that resonance frequency increases as the value of mass attached on CNT decreases as shown in Tables .3 and 4. This satisfies the mass-frequency relationship as established in Eqs. (4 and 8).

Table 3: Resonance frequency of cantilevered CNT with varying mass for three different lengths

Mass (zg)	Frequency		
	L= 6 nm	L= 8 nm	L= 10 nm
0.5	1.47212E+11	8.51724E+10	5.57766E+10
0.05	1.55230E+11	9.04283E+10	5.70061E+10
0.01	1.67694E+11	9.51775E+10	6.14372E+10
0.005	1.77694E+11	9.61775E+10	6.24372E+10

Table .4Resonance frequency of bridged CNT with varying mass for three different lengths

Mass (zg)	Frequency		
	L= 6 nm	L= 8 nm	L= 10 nm
0.5	7.97861E+11	4.83475E+11	3.32431E+11
0.05	7.72185E+11	5.00670E+11	3.45557E+11
0.01	8.02884E+11	5.04272E+11	3.47018E+11
0.005	8.12824E+11	5.09272E+11	3.55018E+11

The variation in resonance frequency with mass for three different lengths has been plotted in Figure .5 for cantilever and in Figure .6 for bridged configuration.

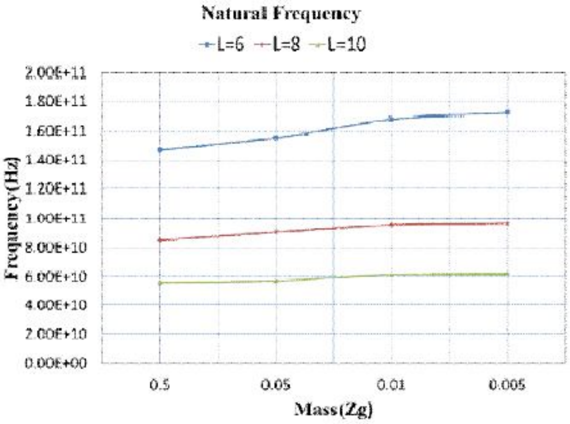


Figure 5: Variation in resonance frequency with mass for Cantilevered CNTs of length 6, 8, 10nm

Form Figures .5 and .6 it has been observed that resonance frequency for bridged configuration is higher, thus indicating more sensitive as compared to cantilever configuration

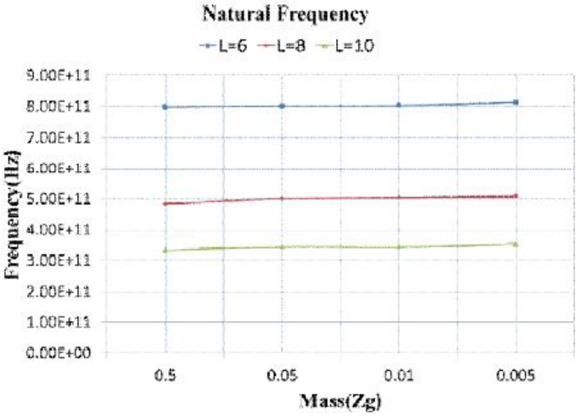


Figure 6: Variation in resonance frequency with mass for bridged CNTs of length 6, 8, 10nm

Further, Figures .5 and .6 clearly suggests that shorter nanotube is highly sensitive for mass sensing and due to decrease in value of attached mass the resonance frequency increases.

CONCLUSIONS

The main characteristics that make carbon nanotubes (CNTs) a promising technology for many future applications are: extremely high strength, low mass density, linear elastic behaviour, almost perfect geometrical structure, and nanometre scale structure. Also, CNTs can conduct better electricity compare to copper and good transmit heat compare to diamonds. Therefore, they are bound to find a wide and possibly revolutionary use in all fields of engineering. The present work analyses the natural frequency of carbon nanotubes with mass variation under cantilevered and bridged conditions. It has been observed that resonance frequency decreases with increase in value of attached mass. Further, from simulation results indicates that shorter length of CNT is more sensitive towards mass detection. This is because of the fact that due to increase in length of CNT its resonance frequency decreases as clear from Tables .3 and .4. Thus it can be concluded that present model can be used for designing of mass sensor specially in the areas where it is required to detect mass as small as of the order of zeptogram level.

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