DISCRETE SLIDING MODE CONTROL FOR THE LATERAL DYNAMICS OF A UAV WITH MINIMUM CONTROL SURFACES

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Abstract— With the evolution of high power density battery, cheap air frames, powerful microprocessors etc. Unmanned aerial vehicle (UAV) started playing a great role in various areas like traffic monitoring, aerial surveillance, hurricane hunting and so on. The UAV's used for these purposes are usually flown at low altitudes, almost below 1000m. Flying at such low altitudes make the UAV easy to crash. Therefore a robust and accurate autopilot system is necessary for small UAVs. The objective of this paper is to investigate the feasibility of discrete sliding mode approach for the lateral dynamics of an unmanned aerial vehicle, P15035. The controller is designed for a linear SIMO model.

Keywords— Discrete Sliding Mode Control, Elevons, Model Reference Sliding Mode Control, Quasi Sliding Mode.

I. INTRODUCTION

Recent advances in communications, solid state devices, and battery technology have made small, low-cost fixed wing unmanned air vehicles (UAVs) a feasible solution for many applications in the scientific, civil and military sectors.With the use of on-board cameras this technology can provide important information for low-altitude and high resolution applications such as scientific data gathering, surveillance for law enforcement and homeland security, precision agriculture, forest fire monitoring, geological survey, and military reconnaissance. Unlike the more familiar flying wings, the aircraft P15035 (Fig.1) belongs to the class of aircraft called flying wings and is known colloquially as a plank having an unswept constant chord (width) wing of low aspect ratio. With the propeller mounted in a tractor configuration but having no rudder or elevators, its attitude is controlled by two independently driven elevon surfaces. A number of advantages have been claimed for flying wings including reduced parasitic drag due to the absence of an extended tail, associated elevator and rudder control surfaces. The specifications of P15035 are given in table 1. The P15035 has two elevon control surfaces which combine the functions of elevators and ailerons. Pitch is controlled by the average deflection of the elevons and roll by the difference of left and right angle deflections. While there is a vertical stabilizer, it has no attached rudder and so yaw control is indirect through roll. The aircraft does not have the usually long moment arm provided by elevators; it must rely upon a slight upsweep in the rear of airfoil to maintain a positive pitching moment to overcome the moments introduced by a forward center of gravity this being essential to maintain stability. Partially as a consequence of this, there is an increased coupling between throttle and pitch. In this study the throttle setting is assumed to be constant. The dynamics of majority fixed-wing aircraft exhibit nonlinearity and strong coupling among state variables that are subject to uncertainties caused by changes of flying conditions and unpredictable turbulence.

Fig.1. P15035

The preliminary work in this field was identification of UAV dynamics [2], in which it was established that lateral and longitudinal dynamics can be decoupled. Proportional-integral-derivative (PID) based autopilot was discussed in [3], which gave satisfactory results after field tuning. The field tuning of PID autopilots is difficult especially when payload varies and in presence of uncertainties. In the model-based control system, mainly the observer-based ones, there always exist uncertainty and modeling errors. Since the identified model is used to design the real-time control systems, consequently, this will degrade robustness of the closed-loop control system. In worst case, the design results may not work practically due to lack of robustness. Sliding mode controller which is a robust technique can provide better performances than PID for nonlinear systems especially when uncertainties are involved [7]. In the actual systems, controllers are implemented in the discrete-time domain since they use microprocessors or computers in general. Discrete time SMC controllers are implemented in many nonlinear systems [7], [8], [10]. The design and stability analysis of different nonlinear systems are described thoroughly in [1], [5], [6], [9].

Table1: Specifications of aircraft P15035

Span	150cm	Chord	35cm
Length	106cm	Control surface	Elevon
Weight	2.9-4.6kg	Motor	Electric
Speed	33-150 km/h	Battery	28×GP3300 NiMH
Duration	40-60 minutes	Autopilot	MP2028

The organization of this paper is as follows. Section II contains P15035's special features and associated modeling. Section III consists of robust sliding mode autopilot for lateral dynamics of UAV. Model reference sliding mode for tracking is proposed in section IV. Simulations and results are presented in section V while conclusion in VI.

II. OPEN-LOOP MATHEMATICAL MODEL

Analyzing the nonlinear model for the aircraft is usually impractical. Instead, a more realistic approach is to develop a set of linearized models valid for different dynamic ranges. Longitudinal and lateral model for conventional aircrafts is studied [2], [4], [11], [12]. It is found that the lateral dynamics is to be uncoupled from its longitudinal motions. Pitch is controlled by the average deflection of the elevons, meanwhile, roll is controlled by the elevon deflections in an attempt to control yaw and to minimize the adverse drag. The longitudinal and lateral directional models for P15035 obtained in [4], [11], [2], [12] is used. Consider the trimmed model in which the throttle is constant, two controllable inputs of the model are right elevon, left elevon and the three outputs are given by output state vector $[q\ p\ r]^T$, representing rates of pitch, roll and yaw vectors, respectively. Hence, the linear trimmed model of our flying wing UAV [2] can be depicted as follows:

$$\begin{bmatrix} q \\ p \\ r \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \\ G_{31} & G_{32} \end{bmatrix} \begin{bmatrix} \delta_L \\ \delta_R \end{bmatrix} \quad (1)$$

Where δ_L and δ_R represent left and right elevons (degree) and G_{ij} are the corresponding transfer functions. Due to the symmetrical properties of the aircraft about its x-z plane, consequently, the effects of left and right elevons are identical to the pitch but opposite to the roll and yaw, therefore $G_{11}=G_{12}$, $G_{21}=-G_{22}$ and $G_{31}=-G_{32}$. Equation (1) can be written as:

$$\begin{bmatrix} q \\ p \\ r \end{bmatrix} = \begin{bmatrix} G_q & 0 \\ 0 & G_p \\ 0 & G_r \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_D \end{bmatrix} \quad (2)$$

Where $G_q=2G_{11}$, $G_p=G_{21}$, $G_r=G_{31}$, $\delta_A=(\delta_L+\delta_R)/2$, and $\delta_D=\delta_L-\delta_R$ are the average and difference of two elevon deflections, respectively. From (2), it is observed that the longitudinal and lateral dynamics are decoupled, where pitch is independently controlled by elevon average deflection, δ_A corresponding to the elevators of a conventional aircraft, roll and yaw are both driven by elevon difference, δ_D , corresponding to the aileron and rudder for a conventional aircraft. Therefore, no decoupling can be made between yaw and roll due to special configuration of the aircraft. For trimmed flight with a constant engine thrust, the discrete lateral-directional models sampled at 5 Hz are obtained from [2] as:

$$\frac{\phi(z)}{\delta_D(z)} = \frac{.177z^2(z-.9044)}{(z-.912)(z-.9981)(z^2-.629z+.1441)} \quad (3)$$

$$\frac{\psi(z)}{\delta_D(z)} = \frac{-.056z^2(z-1.25)}{(z-.912)(z-.9981)(z^2-.629z+.1441)} \quad (4)$$

From (3) and (4) the linearized model of the lateral dynamics expressed in discrete state space form is given by:

$$\begin{aligned} \mathbf{x}(k+1) &= A\mathbf{x}(k) + \mathbf{b}u(k) \\ y(k) &= C\mathbf{x}(k) + Du(k) \end{aligned} \quad (5)$$

in which, $u(k)=\delta_D$, $y(k)=[\phi(z)\ \psi(z)]^T$ and $\mathbf{x}(k)$ is the state vector defined accordingly:

$$A = \begin{bmatrix} 2.5391 & -2.2558 & .8478 & -.1312 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} .177 & -.16 & 0 & 0 \\ -.056 & .07 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

III. DISCRETE SLIDING MODE CONTROLLER DESIGN

A discrete-time version of sliding mode control is important when control is realized by computers with a relatively slow sampling period. Consider the following single input discrete-time system:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + \mathbf{b}u(k) \quad (6)$$

$\mathbf{x}(k)$, the state vector, $u(k)$, the control input, A and $\mathbf{b}$ are system and input matrices of appropriate dimensions, respectively. When a Discrete sliding mode controller (DSMC) is applied to this system, the state response of the system can be separated into the Reaching Mode (RM), Sliding Mode (SM), and Steady-State (SS) modes. The desired state trajectory of a discrete Variable structure control (VSC) system should have the following attributes:

a. Starting from any initial state, the trajectory moves monotonically towards the switching plane and cross it in finite time.

b. Once the trajectory crosses the switching plane the first time, it crosses the plane again in every successive sampling period, resulting in a zigzag motion about the switching plane.

c. The size of each successive zigzagging step is non-increasing and the trajectory stays within a specified band called quasi-sliding mode (QSM) band.

3.1. SWITCHING SURFACE DESIGN

The design for a desired sliding mode is a technique through which a linear switching function is determined.

$$s(k) = C^T\mathbf{x}(k) \quad (7)$$

Consider a linear switching plane

$$s(k) = C^T\mathbf{x}(k) = 0 \quad (8)$$

The ideal quasi-sliding mode satisfies

$$s(k+1) = s(k) = 0 \qquad , k = 0,1,2 \quad (9)$$

From (6), (7), (8)

$$C^T A\mathbf{x}(k) + C^T \mathbf{b}u(k) = s(k) = 0 \tag{10}$$

Solving for u, an equivalent control is given by

$$u_{eq} = -(C^T\mathbf{b})^{-1} C^T A\mathbf{x}(k) \tag{11}$$

where $C^T\boldsymbol{b} \neq 0$ has been assumed, implying the controllability of the VSC system. The control (10) is linear in **x**, so dynamical equation of the ideal quasi-sliding mode is also linear, given by

$$\mathbf{x}(k+1) = [I - \mathbf{b}(C^T\mathbf{b})^{-1} C^T A\mathbf{x}(k)] \tag{12}$$

Instead of using (11), there is an alternative and more direct approach to analyze the sliding mode dynamics. This approach also provides a way for designing the vector c of the switching function. The approach requires transforming system (5) to a normal form. Let the system (5) be in a normal form in which

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ x_2 \end{bmatrix} \ \mathbf{b}^T = [0,..0,1]$$

$$C^T = \begin{bmatrix} C^T & 1 \end{bmatrix}$$

where A_{22} and x_2 are scalars. Then, when the dynamics of (6) are restricted on the surface, $s(k)=0$, it can be expressed as:

$$\mathbf{x}_1(k+1) = A_{11}\mathbf{x}_1(k) + A_{12}x_2(k) \tag{13}$$

$$x_2(k) = -C^T \mathbf{x}_1(k) \tag{14}$$

Eliminating x_2 gives

$$\mathbf{x}_1(k+1) = \begin{bmatrix} A_{11} - A_{12} & C^T \end{bmatrix} \mathbf{x}_1(k) \tag{15}$$

which is the equation of ideal quasi-sliding mode . It is well known that if (A,$\boldsymbol{b}$) is controllable, then (A_{11} ,A_{12})is also controllable. Under this condition there exists a vector, C such that the eigenvalues of [A_{11}-A_{12} C^T] can be arbitrarily assigned. As a consequence stability of the ideal quasi-sliding mode (15) is guaranteed. Thus the linear switching function is designed such that the sliding mode is stable.

3.2. CONTROL LAW FOR DISCRETE VSC

Instead of first establishing an analytic expression of a reaching condition and then designing a control law to meet the condition a different approach is adopted here. This approach is called the reaching law approach which has been proposed for continuous VSC systems [1], [8], [11]. In this approach, a reaching law is first specified in such a way that all the three attributes specified in section 3 are always satisfied. This reaching law directly dictates the dynamics of the switching function $s(k)$. Then, a VSC control law is synthesized from the reaching law in conjunction with a known model of the plant and the known bounds of perturbations. For the VSC of a continuous plant, a convenient reaching law is

$$\dot{s}(t) = -\varepsilon \mathrm{sgn}(s(t)) - q s(t); \quad \varepsilon > 0; q > 0 \tag{16}$$

For the VSC of a discrete system, an equivalent form of the reaching law is

$$s(k+1) - s(k) = -q\tau s(k) - \varepsilon\tau \mathrm{sgn}(s(k));$$

$$\varepsilon > 0; q\tau > 0; 1 - q\tau > 0 \tag{17}$$

where τ>0 is the sampling period. The inequality for τ must hold to guarantee (3.a), which implies that the choice of τ is restricted. The Presence of the signum term guarantees attributes (3.b) and (3.c). The reaching law (16) always satisfies the reaching condition. Therefore, with a stable ideal quasi-sliding mode, the discrete VSC system designed using the reaching law approach is always stable. The incremental change of $s(k)$ which is

$$s(k+1) - s(k) = C^T \mathbf{x}(k+1) - C^T \mathbf{x}(k)$$

$$= C^T A\mathbf{x}(k) + C^T \mathbf{b}u(k) - C^T \mathbf{x}(k) \tag{18}$$

Solving for $u(k)$ gives the control law

$$u(k) = -(C^T\mathbf{b})^{-1}[C^T A\mathbf{x}(k) - C^T\mathbf{x}(k) + q\tau C^T\mathbf{x}(k) + \varepsilon\tau\, \mathrm{sgn}(C^T\mathbf{x}(k))] \tag{19}$$

IV. MODEL REFERENCE SLIDING MODE CONTROL FOR THE LATERAL DYNAMICS

Tracking problems are often encountered in practice. Consider for example the cases where a robot needs to follow a given trajectory, a machine tool which needs to perform certain prescribed actions, or an airplane which needs to follow a certain flight plan. In all these applications, a controller is required which gives the best possible tracking of the given reference trajectory despite any modelling errors or external disturbances. The two typical cases of tracking control:

1. Target tracking control: For target tracking we assume that some desired output- or state trajectory is defined which needs to be replicated exactly by the system. Consequently, the goal for the closed-loop system is perfect tracking of the desired trajectory.
2. Model reference control: Using a model reference controller we do not aim at perfect tracking of some desired trajectory, but we aim at tracking some ideal model.

The objective of model reference control is to obtain the desired closed-loop behaviour. This desired closed-loop behaviour is defined by a reference model from which the name model reference control originates. The reference model is defined by:

$$\mathbf{w}[k+1] = G\mathbf{w}[k] + Hr[k] \tag{20}$$

where $w \epsilon R^n$, $r \epsilon R^q$, $G \epsilon R^{n \times n}$ and $H \epsilon R^{n \times q}$. Furthermore, it is assumed that the pair *(G, H)* is controllable and the matrix G has stable eigenvalues, hence the reference system is stable. A controller is designed which results in perfect tracking by the original system of the reference state $w[k]$. The following criteria should be satisfied for the reference model *(G, H)* to be tractable by the original system.

Theorem 1: The state $w[k]$ generated by the reference model (20), is tractable by the system (6) if, and only if, the matrices *(A-G)* and H can be written as:

$$A - G = \mathbf{b}\delta_{AG}$$
$$H = b\delta_H \quad (21)$$

for some finite $\delta_{AG} \epsilon R^{m \times n}$ and $\delta_H \epsilon R^{m \times q}$.

Proof: The tracking error e_{xw} can be defined as:

$$e_{xw}[k] = \mathbf{x}[k] - \mathbf{w}[k] \quad (22)$$

Using equations (6), (20), and (22) we can determine the error dynamics to be:

$$e_{xw}[k+1] = Ae_{xw}[k] + (A - G)\mathbf{w}[k] - Hr[k] + \mathbf{b}u[k] \quad (23)$$

If condition (21) is fulfilled, equation (23) can be written as:

$$e_{xw}[k+1] = Ae_{xw}[k] + \mathbf{b}(\delta_{AG}\mathbf{w}[k] - \delta_H r[k] + u[k]) \quad (24)$$

By the transformation $T_{rx} \epsilon R^{n \times n}$, given by:

$$\begin{bmatrix} \bar{e}_{xw,1}(k) \\ \bar{e}_{xw,2}(k) \end{bmatrix} = T_{r_x} e_{xw}(k) \quad (25)$$

The error system (24) into the regular form:

$$\begin{bmatrix} \bar{e}_{xw,1}(k+1) \\ \bar{e}_{xw,2}(k+1) \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{e}_{xw,1}(k) \\ \bar{e}_{xw,2}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} (u[k] - \delta_{AG}\mathbf{w}[k] - \delta_H r[k]) \quad (26)$$

Assuming that at some initial time k_0 the state error is zero, i.e. $\bar{e}_{xw,1}[k_0]=0$ and $\bar{e}_{xw,2}[k_0]=0$, it can be seen from equation (26) that it is sufficient to choose for $u[k] \forall k \geqslant 0$

$$u[k] = \delta_H r[k] - \delta_{AG}\mathbf{w}[k] \quad (27)$$

For the case that condition (21) is not fulfilled, applying the transformation T_{rx} to equation (23) leads to:

$$\begin{bmatrix} \bar{e}_{xw,1}(k+1) \\ \bar{e}_{xw,2}(k+1) \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{e}_{xw,1}(k) \\ \bar{e}_{xw,2}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} (u[k]) + \begin{bmatrix} T_{r_x,1}(A-G) \\ T_{r_x,2}(A-G) \end{bmatrix} \mathbf{w}[k] - \begin{bmatrix} T_{r_x,1}H \\ T_{r_x,2}H \end{bmatrix} r[k] \quad (28)$$

where T_{rx} has been partitioned as

$$\begin{bmatrix} T_{r_x 1}H \\ T_{r_x 2}H \end{bmatrix} \quad (29)$$

With $T_{rx,1} \epsilon R^{(n-m) \times n}$ and $T_{rx,2} \epsilon R^{m \times n}$. Since condition (21) is not fulfilled, either $T_{rx,1}(A\text{-}G)$ or $T_{rx,1}H$ (or both) are nonzero. Therefore, there do not exist a control input which cancels all terms driving the error dynamics which completes the proof. Theorem 1 shows that the matrices *(A-G)* and *H* should both be matched with the input matrix B. A possible interpretation is to see the signals *(A-G)**w**[k]* and *Hr[k]* as measurable disturbances working on the error system.

4.1. STATE-BASED MODEL REFERENCE SLIDING MODE CONTROL

In this section a SDSMC is defined which forces the system (6) to track the reference model (20). It is assumed that condition (21) is fulfilled. Consequently the error dynamics in the regular form are given by (see Theorem 1):

$$\begin{bmatrix} \bar{e}_{xw,1}(k+1) \\ \bar{e}_{xw,2}(k+1) \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{e}_{xw,1}(k) \\ \bar{e}_{xw,2}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} (u[k] - \delta_{AG}\mathbf{w}[k] - \delta_H r[k]) \quad (30)$$

The switching function is defined as:

$$\sigma_{xw}[k] = S(\mathbf{x}[k] - \mathbf{w}[k]) = \bar{S}_1 \bar{e}_{xw,1}[k] + \bar{S}_2 \bar{e}_{xw,2}[k] \quad (31)$$

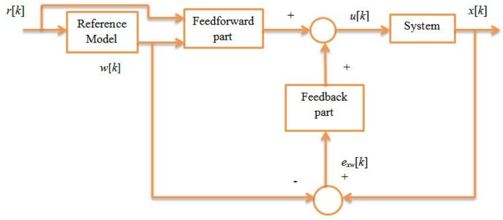

Fig 2: Block diagram of the state-based model reference sliding mode controller.

Then, if we assume that the system is driven into the sliding mode, i.e. $\sigma_{xw}(k)$=0, equation (31) can be written as:

$$\bar{e}_{xw,2}[k] = -\bar{S}_2^{-1}\bar{S}_1\bar{e}_{xw,1}[k] \quad (32)$$

Inserting equation (32) into the representation for $\bar{e}_{xw,1}[k]$, which can be found from equation (30), leads to the reduced order dynamics in the sliding mode:

$$\bar{e}_{xw,1}[k+1] = (\bar{A}_{11} - \bar{A}_{12}\bar{S}_2^{-1}\bar{S}_1)\bar{e}_{xw,1}[k] \quad (33)$$

Again, the matrices $\bar{S}_1$ and $\bar{S}_2$ should be chosen such that the above reduced order dynamics are stable. Interesting to notice is that the design procedure for the sliding surface is again identical to the design of the sliding surface for a stabilizing state based DSMC. The next step is the design of the controller which forces the closed-loop system in the sliding mode. Therefore, we define the linear reaching law as:

$$\sigma_{xw}[k+1] = \phi_{xw}[k] \quad (34)$$

where $\phi \epsilon R^{m \times m}$ being diagonal with all its diagonal entries ϕ_i, i=1…m satisfying $0 \leqslant \phi_i < 1$.From equations (6), (20) and (34) we can determine the control law to be:

$$u[k] = (Sb)^{-1}(S(G-A)w[k] + SHr[k]) + (Sb)^{-1}(\phi\sigma_{xw}[k] - SAe_{xw}[k]) \quad (35)$$

The first part of equation (35) is purely based on off-line computed variables ($r[k]$and $w[k]$) and hence can be considered as a feedforward control signal. The second part of equation (35) is based on the measured state error and hence can be considered as the feedback component of the controller. This can be seen in the block diagram presented in Figure 2.

V. SIMULATIONS

5.1. RESPONSES OF A NOMINAL LATERAL SYSTEM

For simulating the nominal model, system and input matrices given in section 2 are transformed and corresponding values are used. Simulation parameters used are q=.4 and ε=.3. These parameters are tuned so as to obtain the best system response. The quasi-sliding mode (QSM) band is the region in the phase space where the sign of *s(k)* begins to alternate at every iteration. The width of the QSM band

$$2\Delta = \frac{2\varepsilon\tau}{1-q\tau} = .13$$

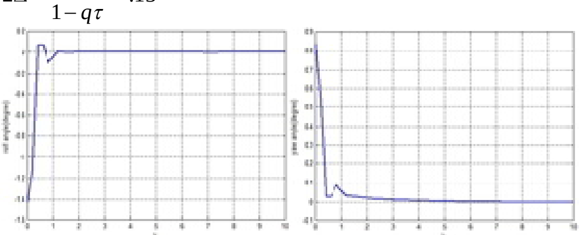

Fig 3: left: Roll angle stabilization for a nominal system using DSMC. Right: Yaw angle stabilization for a nominal system using DSMC.

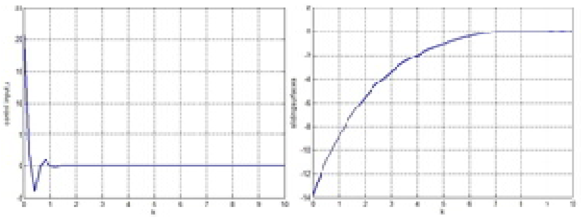

Fig 4: left: Control input. Right: Sliding surface

5.2 STATE-BASED MODEL REFERENCE SLIDING MODE CONTROL FOR LATERAL DYNAMICS

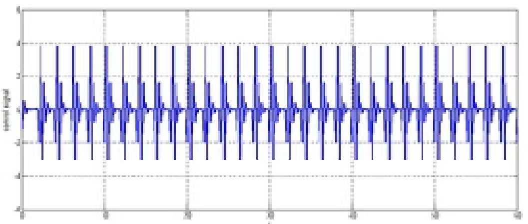

Fig 5: Control input for model reference control

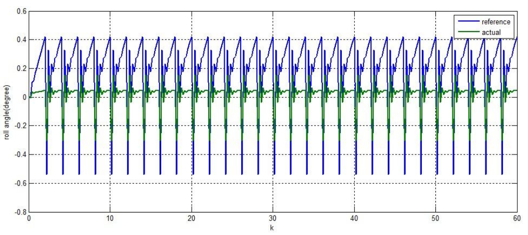

Fig 6: Roll angle tracking using model reference control

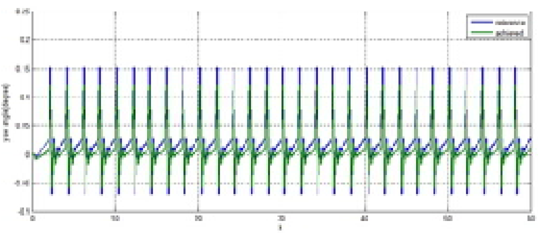

Fig 7: Yaw angle tracking using model reference control

CONCLUSIONS

Discrete time sliding mode controller is found to be efficient in stabilizing the dynamics and in implementing model reference control. is used for tracking the lateral dynamics. Fig (5 & 6) blue line indicates the reference signal to be tracked and green line indicates the achieved signal. Thus a satisfactory performance is achieved. It is well known that high-frequency chattering exists in any continuous VSC system. But it does not exist in a discrete VSC system in general, especially when the sampling rate of the system is low [9]. This is because that, for a discrete VSC system, a quasi-sliding mode band around the switching plane always exists. This work can be extended to investigate the robustness of sliding mode controller in the event of disturbances.

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