MULTI-RESPONSE OPTIMIZATION OF AL2024/RED MUD MMC USING PRINCIPAL COMPONENT ANALYSIS

[1]AMIT SHARMA, [2]R.M. BELOKAR, [3]SANJEEV KUMAR

[1,2,3]PEC University of Technology, Chandigarh, India
E-mail: [1]amiter84@gmail.com, [2]rmbelokar@pec.ac.in, [3]sanjeevkumar@pec.ac.in

Abstract— In this paper stir casting process parameters of the developed Metal Matrix Composite are optimized using Principal Component Analysis (PCA). Aluminium alloy 2024 is used as the matrix material and red mud is used as the reinforcement. Taguchi's L9 orthogonal array is used to design the experiment. Three parameters are considered for optimization process namely reinforcement percentage, grain size and blade angle. Response variables considered for the optimization process are tensile strength and microhardness. ANOVA is applied to find the effect of individual factor on the mechanical properties of the MMC. Results reveal that grain size was the most significant factor followed by reinforcement percentage and blade angle.

Keywords— PCA, Taguchi, ANOVA, AL2024, Red Mud.

I. INTRODUCTION

Metal matrix composite are being widely used in the aerospace and automotive industry because of their enhanced properties like high strength to weight ratio, high modulus, superior wear resistance and corrosion resistance [1]. When performing optimization it is very important to find the right combination of settings that will result in improving more than one property in tandem. Taguchi method optimizes single response which may degrade other quality characteristics so it is very important to optimize all the quality characteristics at the same time [2].

Pearson [3] proposed PCA which was then developed as a statistical tool by Hotelling [4]. Datta et al. utilized PCA for optimizing multi-responses in arc welding process [5]. Adalarasan et al. used GT-PCA approach in optimizing welding parameters and concluded that the optimum settings resulted in improving quality characteristics [6]. Fung and Kang used PCA to improve the friction properties in different sliding directions (P-type and AP-type) [7]. Lu et al. used Grey-PCA to optimize process parameters of high speed end milling of SKD 61 tool steel [8]. Raj developed a surface roughness and delamination mathematical model for prediction of optimum conditions. PCA was proposed to identify the uncorrelated quality indices [9]. Su utilized PCA method to optimize multiple responses and suggests that PCA can be effectively used to identify the optimum conditions [10]. Antony used PCA with Taguchi method for optimizing multiple responses [11].

In this paper Taguchi parametric design and optimization method was used to design the experiments and develop a model to predict the responses as a function of input parameters and the adequacy of the model was checked by conducting the confirmatory tests. Optimization of the problem was done with the help of Principal Component Analysis (PCA) based on Taguchi orthogonal array. ANOVA was performed to identify the significant factors. Results reveal that the CPC value obtained using the optimum settings resulted in improving the quality of the product.

II. EXPERIMENTAL PROCEDURE

2.1Material

In this paper Al2024 is used as the matrix material because of the advantages like good weld-ability, corrosion protection and capability to inhibit stress corrosion cracking. Red Mud is used as the reinforcement and it is widely available free of cost. The elemental composition of Al2024 and red mud are given in table 1 and table 2 respectively.

Table 1: Aluminium Alloy 2024 composition

Conc.	Cu	Mg	Si	Fe	Mn	Zn	Ti	Cr	Al
%	4.29	1.29	0.07	0.2	0.54	0.03	0.06	0.01	Rem.

Table 2: Red Mud composition

Conc.	Al2O3	Fe2O3	SiO2	TiO2	Na2O	CaO	LOI
%	17-19	35-36	7-9	14-16	5-6	3-5	10-12

2.2 Plan of experiment

The processing parameters considered for this study are given in table 1. Degrees of freedom (DOF) needed for designing the experiment is six, so L_9 OA is selected for designing the experiment.

Table 3: Processing parameters and their levels

Parameters	Unit	Levels		
		1	2	3
Percentage	%	5	10	15
Grain Size	Microns	90	150	250
Blade Angle	Degree	90	120	180

2.3 Principal Component Analysis (PCA)

PCA is a multivariate statistical approach introduced by Pearson and further developed by Hotelling. PCA can convert the multiple correlated responses data into several uncorrelated quality indices. A mathematical function is then formulated by gathering all or some quality indices called composite principal component (CPC) which stands for the overall quality of the process. Finally the CPC can be used to determine the optimal conditions. In order to make all the responses with different dimensions at diverse ranges of the system unique, PCA is usually used in the data pre-processing. The procedural steps involved in PCA are given below:

Step 1: Array the measured multiple responses during the process

$$A = \begin{bmatrix} y_{11} y_{12} y_{13} \dots y_{1k} \\ y_{21} y_{22} y_{23} \dots y_{2k} \\ y_{31} y_{32} y_{33} \cdots y_{3k} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ y_{i1}\ y_{i2}\ y_{i3} \cdots y_{ik} \end{bmatrix}$$

Where , i is the number of experimental runs,
k is the number of response,

Table 4: L9 Orthogonal Array along with response variable

Trial No.	Factors			Response	
	Reinforcement	Grain Size	Blade Angle	Tensile Strength (MPa)	Microhardness (VHN)
1	3	3	1	93.57	67.7
2	1	1	1	168.92	70
3	3	1	2	169.39	82.3
4	2	3	2	81.82	68.64
5	3	2	3	129.14	73.3
6	2	1	3	135.33	71.7
7	1	2	2	119.48	69
8	1	3	3	84.73	58.7
9	2	2	1	138.7	72.77

Step 2: Normalize the multiple responses array

The responses are normalized using the following criterion

- Lower the better

$$x_i(k) = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)}$$

- Higher the better

$$x_i(k) = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)}$$

Where

$x_i(k) is\ the\ normalized\ value\ of\ k^{th}\ response$,

$\min y_i(k) is\ the\ smallest\ value\ of\ y_i(k) for\ k^{th} respon$
,

$\max y_i(k) is\ the\ largest\ value\ of \min y_i(k)\ and$
x is the normalized array

$$x = \begin{bmatrix} x_{11} x_{12} x_{13} \dots x_{1k} \\ x_{21} x_{22} x_{23} \dots x_{2k} \\ x_{31} x_{32} x_{33} \cdots x_{3k} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ x_{i1}\ x_{i2}\ x_{i3} \cdots x_{ik} \end{bmatrix}$$

Table 5: Normalised Value of the responses

Experimental run	Normalized Values	
	Tensile Strength	Microhardness
Reference Sequence	1	1
1	0.134178372	0.381355932
2	0.994632865	0.478813559
3	1	1
4	0	0.421186441
5	0.540367706	0.618644068
6	0.611054014	0.550847458
7	0.430055955	0.436440678
8	0.033230558	0
9	0.649537513	0.596186441

Step 3: Calculate the variance-covariance matrix M from the normalized data

$$M = \begin{bmatrix} N_{11} N_{12} N_{13} \dots N_{1k} \\ N_{21} N_{22} N_{23} \dots N_{2k} \\ N_{31} N_{32} N_{33} \cdots N_{3k} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ N_{i1}\ N_{i2}\ N_{i3} \cdots N_{ik} \end{bmatrix}$$

$$N_{k,l} = \frac{Cov[x_i(k), x_i(l)]}{\sqrt{Var[x_i(k)] \times Var[x_i(l)]}}$$

Where l=1,2,3,....,k and $Cov[x_i(k), x_i(l)]$ is the covariance of sequence $x_i(k)$ and $x_i(l)$

Step 4: Calculate the Eigen values and Eigen vectors from the correlation coefficient array and denoted by λ_j and V_j respectively

$$(R - \lambda_j I_m)V_{ij} = 0$$

Where λ_j $eigenvalues, \sum_{j=1}^{n} \lambda_j = n, j = 1,2,3, \dots, n;$
$V_{ij} = [a_{j1} a_{j2} \dots a_{jn}]^T$
$eigenvectors\ corresponding\ to\ the\ eigen\ value\ \lambda_j,$

Table 6: Eigen values and Eigen vectors

Eigen Analysis of the Correlation Matrix		
Variables	PC1	PC2
Eigen Values	1.7393	0.2607
Eigen Vectors	0.707	0.707
	0.707	-0.707
Proportion (%)	0.870	0.130
Cumulative (%)	0.870	1

Step 5: Evaluate the principal components (ψ_j)
The eigen vector V_j represents the weighting factor of k number of quality characteristics of the j^{th} principal component. For example, if Q_j represents the j^{th} quality characteristics, the j^{th} principal component ψ_j will be treated as a quality indicator with the required quality characteristic.

$$\psi_j = \sum_{j=1}^{k} (V_j \times Q_j)^{1/j}$$

Where ψ_1 is the first principal component, ψ_2 is the second principal component and so on. The principal components are aligned in descending order with the respect to variance and therefore the ψ_1 accounts for the most variance in the data.

Table 7: Principal components

Trial No	Principal Components	
	PC1	PC2
Reference Sequence	1.414	0
1	0.3645	-0.1748
2	1.0419	0.3647
3	1.4142	0.0000
4	0.2978	-0.2978
5	0.8195	-0.0553
6	0.8216	0.0426
7	0.6127	-0.0045
8	0.0235	0.0235
9	0.8809	0.0377

Step 6: Evaluate the CPC (ψ)
The CPC (ψ) represents the index of multi-composite quality for multi-quality responses. It is defined as the combination of principal components with their individual Eigen values.

Table 8: CPC values of the responses

Trial No	CPC	Quality Loss
Reference Sequence	1.23	0.00
1	0.29	0.94
2	0.95	0.28
3	1.23	0.00
4	0.22	1.01
5	0.71	0.52
6	0.72	0.51
7	0.53	0.70
8	0.02	1.21
9	0.77	0.46

Table 9: Main effects of the CPC

Response Table for Means			
Level	Reinforcement	Grain Size	Blade Angle
1	0.5033	0.9682	0.6732
2	0.5707	0.6698	0.6611
3	0.7435	0.1794	0.4832
Delta	0.2403	0.7887	0.19
Rank	2	1	3

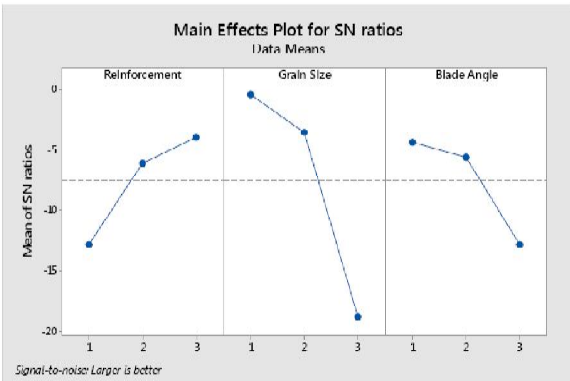

Figure 1: Main effects plot of SN ratios

Table 10: ANOVA results for CPC

Analysis of Variance						
Source	DOF	Seq. SS	Adj. SS	Adj. MS	F-Value	% Contribution
Reinforcement	2	0.09215	0.09215	0.04608	2.3	8 %
Grain Size	2	0.95161	0.95161	0.47581	23.78	82.82 %
Blade Angle	2	0.06787	0.06787	0.03394	1.7	5.89 %
Residual Error	2	0.04002	0.04002	0.02001		3.47 %
Total	8	1.15166				100%

2.4 Confirmation experiment

The confirmation experiment is a crucial step and is highly recommended by Taguchi to verify the experimental conclusions. The purpose of the confirmation experiment in this study is to validate the optimum fabrication conditions that are suggested by the experiment. The optimum conditions are set for the significant factors and the insignificant factors are set at economic level. Selected number of tests is run under constant specified conditions. The average of the results of the confirmation experiment is compared with the anticipated average based on the parameters and levels tested. The estimated mean of the response characteristics is computed. A confidence interval for the predicted mean on a confirmation run is calculated using the equation given below:

$$CI_{CE} = \sqrt{F_{\propto}(1, f_e)V_e\left[\frac{1}{n_{eff}} + \frac{1}{R}\right]} = \pm 0.5379$$

Where $F_{\propto}(1, f_e)$ is the F-ratio at a significance level of α%, α is the risk, f_e is the error degrees of freedom, V_e is the error mean square, n_{eff} is the effective total number of test and R is the number of confirmation tests.

n_{eff}=effective number of replications=N/[1+total DOF in the estimation of mean],

The expected mean at the optimal settings (μ) is calculated by using the following model,

$$\mu_{CPC} = \bar{A}_3 + \bar{B}_1 + \bar{C}_1 - 2\bar{T}_{CPC}$$

Where $\bar{A}_3, \bar{B}_1 \, and \, \bar{C}_1$ are the mean values of the CPC with the parameters at the optimum levels and $\bar{T}_{CPC}$ is the overall mean of CPC. The expected mean is found out to be 1.1732.

Therefore 95% confidence interval of the predicted optimum condition is given by following model,

$0.6353 < \mu_{CPC} < 1.7111$

Table 11: Summary of the result

Response Variable	Final Optimum Parameters	
	Predicted	Experimental
	A3B1C1	A3B1C1
CPC	1.1732	1.2670

CONCLUSION

In this paper PCA is effectively used to find the optimum conditions of the stir casting process parameters to predict the mechanical properties. The application of PCA convert correlated responses into uncorrelated quality indices called principal components which have been used for optimization process. ANOVA results reveal that the most significant factor was found out to be grain size followed by reinforcement percentage and blade angle. The use of PCA resulted in enhancing the quality of the developed Al2024/red mud MMC and the optimum values were found to be 1.267.

REFERENCES

[1] Ding X, Liew WYH, Liu XD (2005), “Evaluation of machining performance of MMC with PCBN and PCD tools”, Wear, 259: 1225-1234.

[2] Palanikumar K, Karunamoothy L, Karthilesyan R (2006), “Multi performance optimization of machining parameters on the machining of GFRP composite using carbide (K-10) tool”, Mater. Manuf. Process, 21:846-852.

[3] Pearson K (1901), “On lines and planes of closest fit to systems of points in space”, Philos. Mag., 62:559-572.

[4] Hotelling H (1993), “Analysis of a complex of statistical variables into principal components”, J Edue Psychol, 24:417, 441.

[5] Datta S, Nandi G, Bandyopaghyay A, Pal PK (2009), “Application of PCA based hybrid Taguchi method for correlated multicriteria optimization of submerged arc weld: a case study”, Int. J. Adv. Manuf. Tech, 45(3-4):276-286.

[6] Adalarasan R, Shanmuga S A (2014), “Parameter design in friction welding of Al/SiC/Al2O3 composite using grey theory based principal component analysis (GT-PCA)”, J Braz. Soc. Mech. Sci. Eng, 37:1515-1528.

[7] Fung C P and Kang P C (2005), “Multi response optimization in friction properties of PBT composites using Taguchi method and principal component analysis”, Journals of Materials Processing Technology, Vol. 170, no. 3, pp. 602-610.

[8] Lu Hs, Chang CK, Hwanga NC, Chung CT (2009), “Grey relational analysis coupled with principal component analysis for optimization design of the cutting parameters in high speed end milling”, J. Mater. Process Technology, 209:3808-3817.

[9] Raj P, Elaya P A, Ramu P (2012), “Prediction of surface roughness and delamination in end milling of GFRP using mathematical model and ANN”, Indian J. Eng. Mater. Sci, 19:107-120.

[10] Su C.T, Tong L.I (1997), “Multi response robust design by principal component analysis”, Total Qual. Manage, 8(6):409-416.

[11] Antony J (2000), “Multi response optimization in industrial experiments using Taguchi’s quality loss function and principal component analysis”, Quality and Reliability Engineering International, 16(1), 3-8.

★★★