ANALYSIS OF SIGNAL DENOISING METHODS BASED ON WAVELET TRANSFORM

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Abstract: The real world signals do not exist without noise. Wavelet Transform based denoising is a powerful method for suppressing noise in signals. In this paper, signal denoising based on Double-Density Discrete Wavelet Transform (DDDWT) and Dual-Tree Discrete Wavelet Transform (DTDWT) methods are implemented with optimum values of threshold point and level of decomposition. Based on the intensity of noise in the received signal, optimum values of threshold point and level of decomposition are determined. The results in terms of Root Mean Square Error (RMSE) and Signal to Noise Ratio (SNR) are then compared with the corresponding values of Discrete Wavelet Transform (DWT) based denoising method. The popular test signal; piece-regular contaminated with Additive White Gaussian Noise (AWGN) is chosen for the implementation. The results of MATLAB simulations show that for the selected threshold point and level of decomposition, the DDDWT and DTDWT perform better than the DWT method.

Keywords: Signal Denoising, Discrete Wavelet Transform, Double- Density Discrete Wavelet Transform, Dual-Tree Discrete Wavelet Transform, Root mean square error.

I. INTRODUCTION

Wavelet transform is a well-known tool that finds applications in the areas of digital signal and image processing. It provides a multi-resolution analysis by representing a function simultaneously in time and frequency domains using a set of analyzing functions called wavelets [1]. This good time-frequency locality of wavelet transform makes it useful for processing of non-stationary and transient signals [2]. The wavelet transform, particularly Discrete Wavelet Transform (DWT) performs well in noise removal applications. As multi-resolution analysis is not possible with other transforms like Fourier Transform and Short Time Fourier Transform, they cannot be applied much in signal denoising applications.

Wavelet transforms are of different types. The critically-sampled form of the wavelet transform provides the most compact representation. But it lacks shift-invariance and directional selectivity. These problems can be avoided by using complex wavelet transform or the extensions of DWT namely, double-density DWT (DDDWT) [3] and dual-tree DWT (DTDWT) [4]. The double-density DWT and the dual- tree DWT are similar in several ways as both of them are based on perfect reconstruction filter banks; they are over complete by a factor of two and are nearly shift invariant [5]. Both wavelet transforms perform well when compared with the critically-sampled DWT for signal denoising applications.

In this paper, signal denoising based on the expansive forms of discrete wavelet transform namely Double-Density Discrete Wavelet Transform (DDDWT) and Dual-Tree Discrete Wavelet Transform (DTDWT) are implemented. The threshold point and the level of decomposition for the wavelet denoising method depend on the noise intensity. Based on the intensity of noise in the received signal, the optimum values of threshold point and level of decomposition are determined experimentally. The performance of the denoising methods is evaluated based on Root Mean Square Error (RMSE) and Signal to Noise Ratio (SNR). A comparative study was also performed to show the effectiveness of the use of DDDWT and DTDWT methods for denoising signals with the DWT method. With a noise intensity of 15 for the received noisy signal, the optimum values of threshold point and level of decomposition were found to be 20 and 4 respectively.

The paper is organized as follows. The introduction is followed by a brief background to wavelet denoising methods in Section 2. Section 3 provides the description of the proposed work, its experimental results and discussions are presented in Section 4 and Section 5 concludes with practical recommendations.

II. WAVELET DENOISING METHODS

The simplest wavelet transform used for processing of digital data is the critically-sampled separable wavelet transform. This is the conventionally used transform and it employs a 1-D wavelet transform in each dimension [7].

A filter bank is an important structure in wavelet transform applications. The analysis and synthesis filter bank for one dimensional filter bank is shown in Fig. 1. The analysis filter bank consists of two filters - a low pass filter, f_1 and a high pass filter, f_2. These filters decompose the input signal x(n) into two subbands. These signals are down-sampled to produce the low frequency and the high frequency parts, c(n) & d(n) respectively. Similarly, the synthesis filter bank consists of two filters- a low pass filter, f_1' and a high pass filter, f_2'.The two subband signals after up-sampling and filtered by these filters are combined to form the reconstructed signal y(n).

The original signal can be reconstructed only if the filters satisfy perfect reconstruction property [8] – [9].

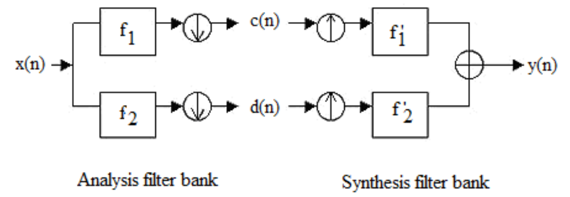

Fig. 1. One dimensional filter bank

2.1. Double-Density Discrete Wavelet Transform

The double-density DWT employs one scaling function and two distinct wavelets. It offers several advantages over critically-sampled DWT. It is shift-invariant and over complete by a factor of two. This transform performs well in the denoising of two dimensional signals. The double density DWT is designed with the filter bank given in Fig. 2 [3].

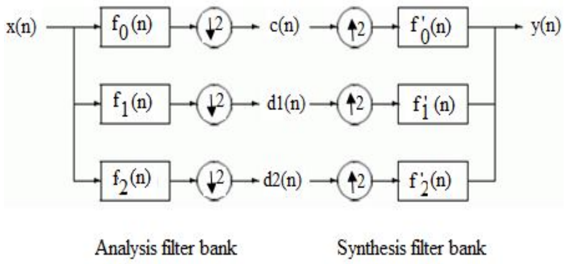

Fig. 2. Double-Density Discrete Wavelet Transform

The analysis filter bank consists of three analysis filters- one low pass filter, $f_o(n)$ and two different high pass filters $f_1(n)$ and $f_2(n)$. These filters decompose the signal x(n) into three subbands. These signals are then down-sampled by 2 to produce the low frequency subband and the two high frequency subbands c(n), $d_1(n)$ and $d_2(n)$, respectively. Similarly, the synthesis filter bank consists of three filters; which are inverse of the analysis filters. The low pass filter is denoted by $f_o'(n)$ and the two high pass filters are denoted by $f_1'(n)$ and $f_2'(n)$. The three subband signals are up-sampled by two, filtered and combined to form the output signal y (n) [10].

2.2. Dual-Tree Discrete Wavelet Transform

Although double-density DWT has several advantages over critically-sampled DWT, some of the wavelets used by double-density DWT lack a dominant spatial orientation which prevents them from being able to isolate those directions. This can be overcome by using dual-tree DWT [6]. It is based on two scaling functions and four distinct wavelets. The wavelets are designed in such a way that the two wavelets of the first pair are offset from one other by one and a half, and the other pair forms a Hilbert transform pair [11]. Improved directional selectivity can then be achieved with dual-tree DWT and it can be used to implement complex and directional wavelet transforms in multiple dimensions [12].

The dual-tree DWT is designed with the filter bank given in Fig. 3. The input signal x(n) is applied simultaneously to two critically-sampled DWTs which form real and imaginary trees A and B respectively. Fig. 3 shows the analysis filter bank with 3 levels of decomposition.

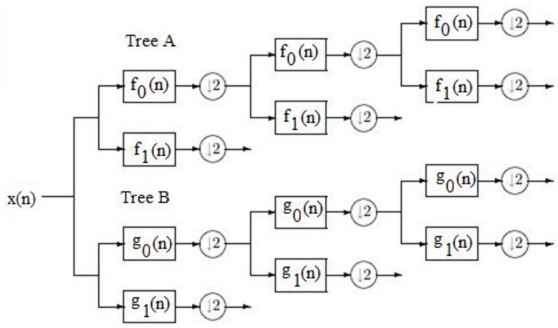

Fig. 3. Dual-Tree Discrete Wavelet Transform

III. PROPOSED WORK

Wavelet Transform denoising method is a powerful method for suppressing noise in signals. The method selected for denoising process depends mainly on the intensity of the noise in the received signal. Based on the intensity of noise in the received signal, the threshold point and level of decomposition for wavelet denoising are selected. The work presented here is for denoising of a signal with noise intensity of 15.

The steps involved for the denoising process are given below.

1. Read the noisy signal y(i). The noisy signal is of the form given in equation 1.

$$y(i) = x(i) + \sigma\varepsilon(i), \quad i = 0,1,2....., n \qquad (1)$$

where y(i) is the signal received - the noisy signal, x(i) is the noise-free signal to be detected and Ɛ(i) is the noise signal with noise intensity σ, and n is the length of the signal [13].

Simulations are carried out on the most popular test signal, piece-regular. One of the best methods to test the effect of noise on a signal is to add Additive White Gaussian Noise, with noise intensity of σ to form the noisy signal.

2. Compute the Root Mean Square Error (RMSE) and Signal to Noise Ratio (SNR) of the noisy signal.

RMSE and SNR can be computed using the formulae given by equations 2 and 3 respectively.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - x_i)^2}{n}} \qquad (2)$$

$$SNR\ (dB) = 10\log_{10}\left[\frac{\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n}(y_i - x_i)^2}\right] \qquad (3)$$

where x and y are the original and the noisy signal respectively, n is the length of the signal.

3. For the observed SNR/RMSE of the received signal, set the level of decomposition and threshold value for wavelet decomposition. The level of decomposition and threshold value depend on the noise intensity.

4. Decompose the received signal into wavelet coefficients using forward double-density DWT/dual-tree DWT. Utilize separate filter bank for all the stages. This step is repeated for different levels. For each level the SNR/RMSE is calculated. The level with high SNR or low RMSE is found to be the optimum level.

5. Threshold wavelet coefficients: Process each subband separately in a loop; use the selected threshold, apply soft thresholding to the decomposed wavelet coefficients through all scales and subbands.

In wavelet transform based denoising method, threshold selection is very important. If the threshold selected is too small or too large, the signal cannot be accurately estimated [14]. The two types of thresholding used in wavelet denoising are given by equations 4 and 5.

Soft threshold: $\{y = sign(x)(|x| - T)$ (4)

Hardthreshold: $\begin{cases} y = x, if\,|x| > T \\ y = 0, if\,|x| < T \end{cases}$ (5)

where x is the input signal, y is the signal after threshold and T is the threshold value [17].

This step is repeated for different threshold points. The threshold value which gives high SNR/low RMSE is found to be the optimum value.
For all the denoising methods, the number of levels of decomposition used is 4 and the threshold selected is 20. These values are experimentally found to be the optimal values for this work and they are presented in the results.

6. Reconstruct the signal: Compute reconstruction using the new wavelet coefficients by calculating inverse double-density DWT/ dual-tree DWT.

7. Compute RMSE and SNR of the reconstructed signal using equations 2 and 3. These values are then compared with the values obtained in step 2 for evaluating the performance. Lower the RMSE or higher the SNR, better is the performance of the denoising method.

Fig. 4 shows the important steps involved in the denoising process.

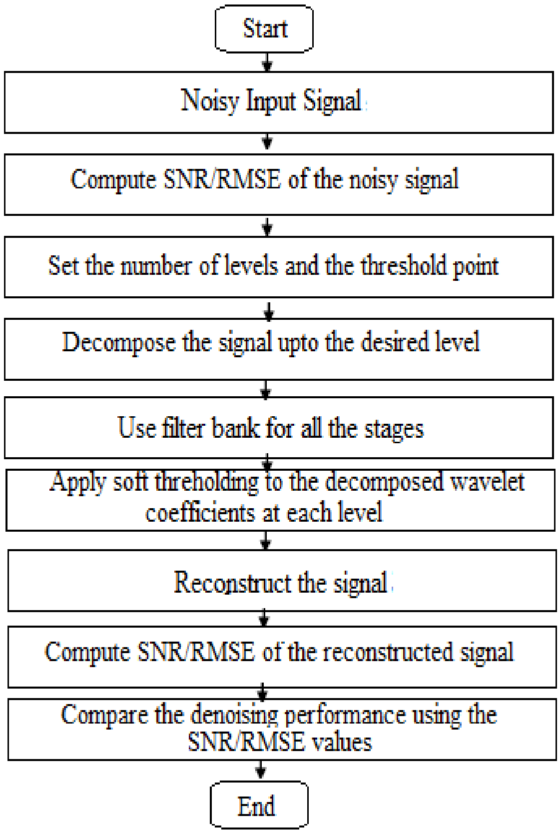

Fig. 4. Work flow of the denoising method

VI. EXPERIMENTAL RESULTS

The above discussed methodology has been implemented in MATLAB 7.10.0 (R2010a). The piece-regular signal is chosen as test signal for the analysis. Additive White Gaussian Noise (AWGN) with noise intensity of σ=15 is added to the signal to form the noisy signal. The SNR of the noisy signal is found to be 24.0632 dB.
Wavelet transform (DWT) denoising method used Daubechies wavelet (db4) at four scales of decomposition. Daubechies wavelet provides a well orthogonality to high frequency noise with a given number of vanishing moments. In wavelet based method, after decomposition of the signal with the chosen wavelet and levels into approximation and detail coefficients, soft thresholding is applied to the detail coefficients. Soft thresholding reduces the sharp changes and provides more visually good reconstructed signals [15].
Table 1 gives the SNR values of all denoising methods for the level of decomposition from 1 to 8 and Fig. 5 gives the plot of SNR against level of decomposition for the different denoising methods. For each method low RMSE or high SNR is preferred for better denoising. There is a limit to extend the number of stages or level of decomposition for the transform. After a certain limit the performance of the system degrades. Therefore, optimal value for number of levels should be selected while removing noise from noisy signals.

Table 1: SNR values of denoising methods for different levels of decomposition SNR of noisy signal = 24.0632dB

Level of Decomposition	SNR (dB)		
	DWT db4 Wavelet, Soft thresholding	DDDWT	DTDWT
1	26.8538	28.5490	27.4653
2	29.9179	31.0656	30.0610
3	31.3888	33.0408	32.1105
4	**33.2718**	**33.5213**	**33.7968**
5	31.8098	32.6594	33.4143
6	29.6179	31.5629	32.9109
7	28.4161	30.7375	32.1054
8	27.7075	30.4477	31.2886

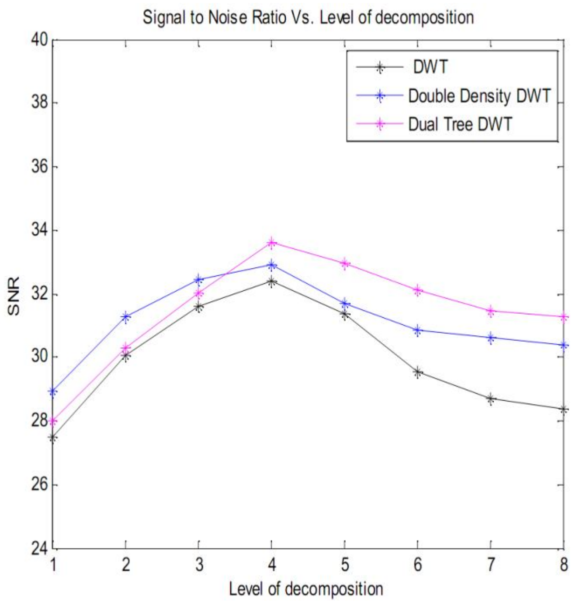

Figure 5. SNR Vs. level of decomposition of denoising methods

From Table 1 and from the graph in Fig. 5, it is clear that level 4 has highest SNR for all the denoising methods, hence level 4 is considered as optimal decomposition level for denoising the signal with SNR of 24.0632 dB.

It is also proved that increasing the levels of decomposition generally increases the computational complexity of the wavelet denoising algorithm and this does not give any reasonable improvement in signal quality too**.** This can be understood from the SNR values of Table 1. Beyond level 4 SNR value decreases.

Table 2 gives the RMSE and SNR values of double-density DWT and dual-tree DWT methods at different threshold points. Figs. 6 and 7 give the plots of RMSE and SNR respectively for different threshold points of noisy signal, double-density DWT and dual- tree DWT denoised signals.

Table 2: RMSE and SNR values of double-density DWT and dual-tree DWT methods for different threshold points

Threshold	RMSE		SNR (dB)	
	Double Density DWT	Dual Tree DWT	Double Density DWT	Dual Tree DWT
0	14.9559	14.9559	24.6069	24.6069
5	10.4738	10.3854	27.7627	27.8363
10	7.3839	7.2532	30.7991	30.9542
15	5.8485	5.7713	32.8239	32.9393
20	**5.4916**	**5.3912**	**33.5213**	**33.7968**
25	5.5781	5.4283	33.2351	33.4715
30	5.6820	5.4911	33.0748	33.3716
35	5.8015	5.5484	32.8940	33.2815
40	5.9041	5.5969	32.7417	33.2067
45	5.9549	5.6489	32.6673	33.1256
50	6.0085	5.6705	32.5895	**33.0924**

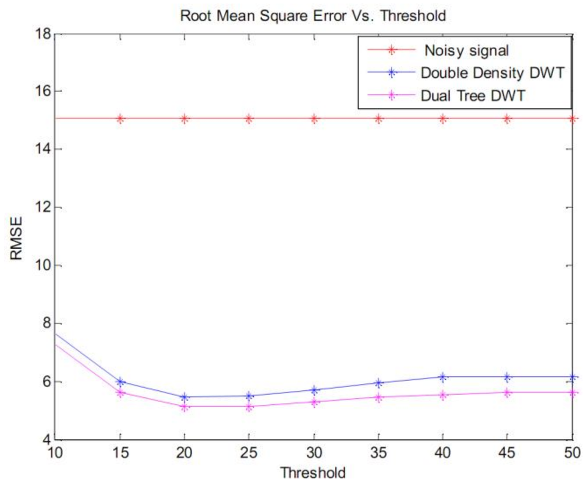

Figure 6. RMSE values of Noisy signal, DDDWT, DTDWT denoised signals for different threshold points

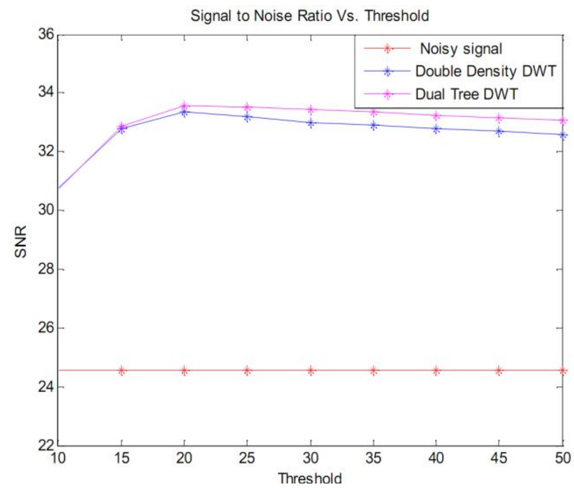

Fig. 7. SNR values of noisy signal, DDDWT, DTDWT denoised signals at different threshold points

From Table 2, we can see that the optimal threshold point is 20. The minimum RMS error or maximum SNR occurs at the optimal threshold point value. At optimal threshold point, we get optimal noise attenuation. From Table 2, it is clear that the DTDWT gives better performance in denoising because RMSE is less and PSNR is more for DTDWT when compared with those for DDDWT. This is also clear from Figs. 6 and 7. The denoising capability of the different methods can be compared, with dual-tree DWT method being the better of the two as it removes more noise than double-density DWT method does. This is achieved at constant values of number of levels, and threshold value for a noise level for both the methods.

Fig. 8 shows the results of different denoising methods applied to piece regular test signal. The original signal, noisy signal, signals denoised by DWT, DDDWT and DTDWT methods are shown. The results are for noise level = 15, threshold point = 20 and number of stages =4.

Table 3 gives the comparison of three methods with the data of noisy signal. Noisy signal with RMSE value of 15.0625 and SNR of 24.0632 is denoised by three methods and we can see the improvement in performance by comparing the RMSE and SNR values. The expansive forms of DWT- DDDWT and DTDWT perform better than the DWT method. Of the three methods with same noise level, 4 stages of decomposition and with a threshold of 20, the dual-tree DWT gives low RMSE and high PSNR values than the other two methods.

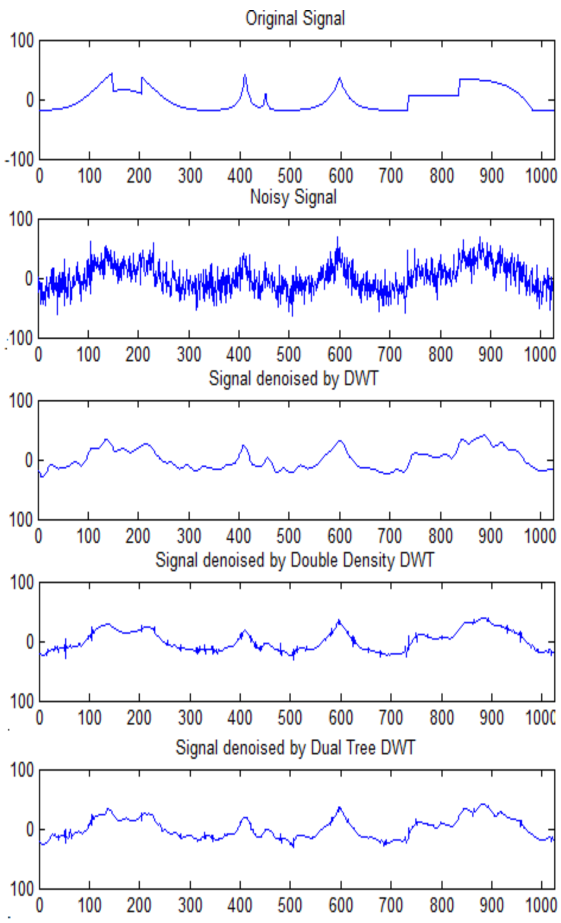

Fig. 8. i) Original Signal ii) Noisy Signal iii) Denoised Signal by DWT iv) Denoised Signal by Double-Density DWT v) Denoised Signal by Dual-Tree DWT

Table 3: RMSE and SNR values for the three methods at threshold value of 20 and noise level of 15

Method	RMSE	SNR(dB)
Noisy Signal	15.0635	24.0632
DWT	6.0551	33.2718
Double Density DWT	5.4916	33.5213
Dual Tree DWT	**5.3912**	**33.7968**

The comparisons of denoising methods for several variations including level of decomposition and threshold points have been made. The results show that most important factor in wavelet denoising is the level of decomposition. It is found out in [16] that the level of decomposition for signal denoising depends also on the frequency band of the signal to be analyzed and its sampling frequency.

CONCLUSION

A lot of research work has been taking place in the areas of signal denoising..Wavelet Transform performs well in this area and the denoising efficiency can be effectively improved by using the expansive forms of DWT. The algorithm developed for implementing the denoising methods in MATLAB is a generalized one which works well with any type of signals. From the results, it is found that the two extensions of the DWT- Dual-Density DWT and Dual-Tree DWT performed well in removing the noise from the input signal for the selected threshold value and the number of levels. These methods give high performance as compared to the existing DWT methods. The results suggest that these methods can be applied to denoise ECG as well as other physiological signals.

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