

# SURVEY ON MULTIPLE USER SPECTRUM SENSING IN THE LOW SNR REGIME (SPHERICAL DETECTOR APPROACH)

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**Abstract**— This paper presents multi-antenna cooperative spectrum sensing in cognitive radio networks, when there are multiple primary users and/or multipath channels. A noise-uncertainty free detector that is optimal in the low signal to noise ratio regime is analysed. We derive the moments of the test statistics involved, which lead to simple and accurate analytical formulae for the key performance metrics. The approximate false alarm and detection probabilities as well as receiver operating characteristic are given in closed form. From the considered simulation settings, performance gain over several known detection algorithms is observed in scenarios with relatively low signal to noise ratio.

We map the problem of multi-sensor spectrum sensing in a multipath multi-primary scenario to a purely multi-primary one. Then we propose the optimal low SNR detector for multi-primary user sensing. Performance analysis of the proposed detection algorithm is addressed.

A spherical test based detection algorithm has been proposed for multi-primary spectrum sensing. Although this detector is the best known multiple primary user detector, it is not the optimal one in the low Signal-to-Noise Ratio (SNR) regime.

**Index Terms**— Signal to Noise Ratio, Cognitive Radio, Multipath-Antenna.

## I. INTRODUCTION

According to survey of Federal Communications Commission (FCC) in 2002, it has been found that spectrum access is more significant problem than physical scarcity of spectrum. With many technological advances in the field of wireless Communication and 3G, 3.5G, 3.75G and 4G technology already being employed Multimedia Broadcast and Multicast Services (MBMS) demand has tremendously increased and with the standardization of MBMS it has gained significant interest in the market. Multimedia content requires more bandwidth, storage capacity and few applications pose tight delay constraints, so the need

to optimize the utilization of spectrum is felt all the more.

## II. COGNITIVE RADIO

Cognitive Radio (CR) is a system/model for wireless communication. It is built on software defined radio which is an emerging technology is providing a platform for flexible radio systems, multiservice, multi-standard, multiband, reconfigurable and reprogrammable by software for Personal Communication Services (PCS). It uses the methodology of sensing and learning from the environment and adapting to statistical variations in real time.

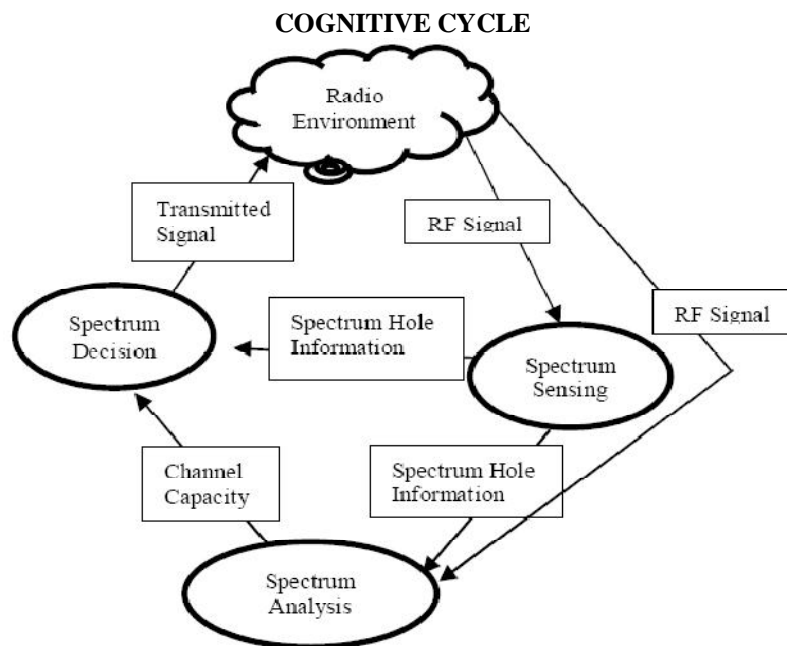


Figure 1: Cognitive Cycle

A basic cognitive cycle comprises of following three basic tasks:

- Spectrum Sensing
- Spectrum Analysis
- Spectrum Decision Making

### III. PROBLEM DEFINITION

In CR networks, dynamic spectrum access is implemented to mitigate spectrum scarcity. A secondary (unlicensed) user is allowed to utilize the spectrum resources when it does not cause intolerable interference to the primary (licensed) user. Recently, there have been changes in the 802.22-2011 standards for Information Technology where there were changes in Cognitive Radio Wireless Regional Area Networks (WRAN), Medium Access Control (MAC) and Physical Layer (PHY) Specifications, where changes were made for changes in policies and procedures for operation in TV bands that allows spectrum sharing where the communication devices may operate in spectrum band of Primary Service.

An expression for the limiting Eigen value ratio distribution is developed, which turns out to be much more accurate than the previous approximations also in the non-asymptotical region. This result is then straightforwardly applied to calculate the decision threshold as a function of a target probability of false alarm. Numerical simulations show that the proposed detection rule provides a substantial performance improvement compared to the other Eigen value-based algorithms.

#### a. SYSTEM DESIGN

In this Paper we consider the standard model for  $K$ -sensor cooperative detection in the presence of  $P$  primary users,

$$x = H_s + \sigma_n \quad (1)$$

where  $x \in \mathcal{C}_K$  is the received data vector,  $k$  – sensors

$H$  is  $K \times P$  matrix =  $[h_1, \dots, h_P]$ ,  $P$  is Primary Users

$P \times 1$  vector  $s = [s_1, \dots, s_P]$  denotes zero mean Transmitted Signal from the primary users.

$\sigma_n$ , the  $K \times 1$  vector is complex Gaussian noise with zero mean and covariance matrix  $\sigma^2 I_K$ ,  $\sigma^2$  is noise power.

Assumptions: -

- 1) The channel  $H$  is constant during sensing time
- 2) The primary user's signal follows a zero mean Gaussian distribution and is uncorrelated with the noise

Due to first assumption the channel model for  $H$  may not be specified, in the absence of PU the sample

covariance matrix  $R = XX$  follows an uncorrelated (white) complex Wishart distribution  $W_K(N, \Sigma)$  with population covariance matrix

$$\Sigma := \frac{E[XX]}{N} = \sigma^2 I_K \quad (2)$$

In presence of Primaryuser, the correlation is given by

$$\Sigma = \sigma^2 I_K + \sum_{i=1}^P \gamma_i h_i h_i' \quad (3)$$

Hence the Rx SNR for Primary user  $i$  across  $K$  sensors is

$$SNR_i = \frac{\gamma_i \|h_i\|^2}{\sigma^2} \quad (4)$$

#### b. TEST STATISTICS

In case of single primary user ( $P=1$ ) the hypothesis test can be expressed as

$$H_0: \Sigma = \sigma^2 I_K \quad (5)$$

$$H_1: \Sigma = \sigma^2 I_K + \gamma_1 h_1 h_1' \quad (6)$$

In case of Multiple primary user scenario ( $P \geq 2$ )

$$H_0 : \Sigma = \sigma^2 I_K \quad (7)$$

$$H_1 : \Sigma > \sigma^2 I_K \quad (8)$$

Deciding the number of primary users is not needed, i.e the hypothesis test is blind in  $P$ , this test is to reject  $H_0$  if we reason out that covariance matrix  $\Sigma$  moves away from sphericity =  $\sigma^2 I_K$ , in statistics this Hypothesis is called sphericity test.

The detection problem can be formulated as –

$H_0$  - Primary Users Absent

$H_1$  - Primary Users Present

Overall, this test is to reject  $H_0$  if we have reason to believe that the population covariance matrix  $\Sigma$  departs from the sphericity =  $\sigma^2 I_K$ . In the statistics literature, this hypothesis test is known as the sphericity test. The test statistics of this Spherical Test based (ST) detector was derived under the GLRT criterion.

$$T_{ST} = \frac{\det(R)}{\left(\frac{1}{K} \text{tr}(R)\right)^K} = \frac{\prod_{i=1}^K \lambda_i}{\left(\frac{1}{K} \sum_{i=1}^K \lambda_i\right)^K} \quad (9)$$

The Likelihood Function of Data Matrix  $X$  is

$$L\left(\frac{X}{\Sigma}\right) = (\det(\Sigma))^{-N} e^{tr(-\Sigma^{-1}R)} \quad (10)$$

The Likelihood ratio statistics is

$$\rho := \frac{\text{Sup}_{\sigma^2 > 0} L(X|\sigma^2 I_K)}{\text{Sup}_{\Sigma > 0} L\left(\frac{X}{\Sigma}\right)} \quad (11)$$

MLE of  $\sigma^2$  under  $H_0$  and  $\Sigma$  under  $H_1$  are

$$\hat{\sigma}^2 = \frac{\text{tr}(R)}{KN} \quad \text{and} \quad \hat{\Sigma} = \frac{R}{N} \quad (12)$$

Inserting (12) into (11) we get

$$\rho^{1/N} = \det \frac{1}{\left(\frac{1}{K} \text{tr}(R)\right)^K} := T_{ST} \quad (13)$$

Assumption  $H_0$  is rejected if  $\rho$  is small i.e when  $\rho^{1/N}$  is small. Hence if  $T_{ST}$  is greater than some threshold value the detector declares  $H_0$  else  $H_1$ .

$$\begin{array}{l} H_0 \\ T_{ST} \geq \zeta \\ H_1 \end{array} \quad (14)$$

Spherical test is formulated as spectrum sensing algorithm.

Firstly, we validate the approx.  $P_{fa}$  and  $P_d$  expressions by Monte Carlo Simulations, then we investigate the Performance of ST detector by comparing with several detection algorithms with and without noise. Parameters K and N are considered in practical spectrum sensing scenario's.

There are two important design parameters for Spectrumsensing:-

1. Probability of False Alarm  $P_{fa}$  – The Probability that secondary user falsely detects Primary signal when primary user is in fact absent

The CDF of  $T_{ST}$  under  $H_0$  can be defined as  $F_{ST}(y)$ . Since  $P_{fa}$  relies on  $F_{ST}(y)$  we observe the characteristics of  $F_{ST}(y)$

CDF for K=2 and K=3 can be obtained as

$$F_{ST}(y) = \frac{2By^{(N-1\frac{2}{3})}\Gamma(N+\frac{1}{2})}{\sqrt{\pi}\Gamma(N-1)}, y \in [0, \infty) \quad (15)$$

For any threshold  $\zeta$  the false alarm probability is obtained as

$$P_{fa}(\zeta) = F_{ST}(\zeta) \quad (16)$$

Hence for any  $P_{fa}$ , threshold can be calculated by numerically inverting  $F_{ST}(\zeta)$

That means -

$$\zeta = \frac{1}{F_{st}(P_{fa})} \quad (17)$$

2. Probability of Detection  $P_d$  – The Probability that Secondary user accurately detects the presence of active primary signal

From (14) probability of detection can be expressed as

$$P_d(\zeta) = G_{ST}(\zeta) \quad (18)$$

If we approximate the parameters to their respective nearest values, then (16) and (18) reduce to simple polynomial equation in  $\zeta$ , thus the threshold will be achieved. For a specific  $P_{fa}$  the resulting threshold is given by  $\zeta$  by (17) this  $\zeta$  can be used to calculate  $P_d$  from (18), the  $P_{fa}/P_d$  mapping is called received operating characteristics.

$$P_d = G_{ST}(F_{ST}^{-1}(P_{fa})) \quad (19)$$

Thus the ROC expression for the ST detection can be obtained as presented by equation no (19).

#### IV. ADVANTAGES

- Useful for high-rate, emerging wireless communication systems such as WiMAX.
- Spectral efficiency.
- Utilization of spectrum

#### CONCLUSIONS

In this article we reviewed that in presence of more than one Primary user, some performance gain may be obtained via the spherical test even without knowing the number of primary users. The need to understand multipath multi-primary user detection with multiple sensors, the results in this direction is rather limited. A spherical test based detection algorithm has been proposed for multi-primary spectrum sensing.

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