AN INVENTORY MODEL FOR PERISHABLE ITEMS WITH INVENTORY DEPENDENT DEMAND RATE, BACKLOGGING AND SHORTAGES IN A CYCLE TIME

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Abstract— In the present model we develop an optimum order quantity inventory model for perishable items with inventory dependent demand rate. Shortages are allowed also. Deterioration of items is permissible. Ordering cost, holding cost, shortage cost, replenishment rate and opportunity cost are available for a cycle time T. A numerical example is also available to study the effect of different parameters.

Keywords— Deterioration, replenishment, backlogging, lead time, holding cost, shortage cost, cycle time.

I. INTRODUCTION

Many researchers have been developed inventory models with and without deterioration. Every item deteriorate time to time which is generated in the nature or manufactured by people like food, chemicals, medicine, paper, wood etc. Many models consider the demand rate to be constant or time dependent but independent form the inventory level. Everyone know that we have limited space to store stock level and can transport very short material. Due to these and more conditions, we have to placed limited and frequent order. So the cost increases very vastly. If we have very large space to store the items then it may be possible to damages of items due to many factors like storage condition, weather condition, insects biting, fungus, auto deterioration etc. Some items are slowly perishable then they can store in huge quantity. But if a product is highly perishable then the cost is more dependent on deterioration and ordering. And seller may prefer to backlog demand in order. But some of customers may accept backlogging during the shortage period, while the others would not. In the practical situations the smaller demand may accepted with shortages but large demand cannot be accepted.

In the first the optimal polices for deteriorating items was made by Ghare and Schrader[6], who revised form of the economic order quantity by EOQ model assuming exponential decay. Covert and Philip[4] developed some inventory model for two parameter weibull distribution for constant demand rate without shortages. Teng, Cheng, and Ouyang[17] presented a model for deteriorating items with power form stock dependent demand. Singh and Shrivastava[16] introduced an EOQ model for perishable items with stock dependent selling rate and permissible delay in payment and partial backlogging. Sharma and Preeti[15] developed optimum ordering interval for random deterioration with selling price and stock dependent demand rate and shortages. Jaggi and Aggarwal[11] have developed an inventory model for credit financing in economic ordering policies of deteriorating items. Jaggi C. K. et al [8] produced a model for pricing and replenishment policies for imperfect quality deteriorating items under inflation and permissible delay in payments. Kumar, Chauhan and Kumar[12] studied a deterministic inventory model for deteriorating items with price dependent demand and time varying holding cost under trade credit. Mo Jiangtao, Chen Guimei, Fan Ting and Mao Hong[14] established optimal ordering policies for perishable multi-item under stock dependent demand and two level trade credits. Jiang Wu, Liang Yuh Ouyang, Leopoldo Eduardo Cardenas Barron and Suresh Kumar Goyal[10] studied an optimal credit period and lot size for deteriorating items with expiration dates under two level trade credit financing. Chang, Goyal, Teng.[3] have presented an EOQ model for perishable items under stock dependent selling rate and time dependent partial backlogging. Jaggi Kausar and Khanna[9] have given an inventory model for Joint optimization of Retailers unit selling price and cycle length under two stage credit policy when the end demand is price as well as credit period sensitive. Sarkar[2] has given an EOQ model with delay in payments and time varying deterioration rate. Bhunia, Pal, Chattopadhyay, Medya[1] have given an inventory model of two warehouse system with variable demand dependent on instantaneous displayed stock and marketing decisions via hybrid RCGA. Yongrui Duan, Guiping Li, James M. Tien and Jiazhen Huo[4] has developed an inventory models for perishable items with inventory level dependent demand rate.

II. ASSUMPTIONS AND NOTATIONS

- Replenishment rate is infinite and lead time is zero.
- C is purchase cost per unit item.
- C_h is inventory carrying cost per unit per unit time.
- A is ordering cost.
- C_s is shortage cost per unit item.
- R unit opportunity cost of lost sales

- Deterioration rate is $\theta = \theta_0+\gamma t$
- Time t_2 for positive and zero inventory and $t_1 < t_2$.
- T is cycle time.
- Demand D(t) = $\begin{cases} \alpha+\beta I(t), & 0\le t\le t_1 \\ \dfrac{\alpha+\beta I(t)}{1+e(T-t)} & t_1\le t\le T \end{cases}$
- α, β, e are positive constant.

III. MATHEMATICAL MODEL

In this inventory model, the inventory does not deteriorate initially from time zero to t_1. The change in inventory level is linearly defined. After this time t_1, the inventory level is due to joint effect of deterioration and demand up to time t_2. Now demand is backlogged in time interval (t_2, T). Under the above conditions the instantaneous inventory level is given by following differential equations

$$\frac{d}{dt}I(t) = -\alpha-\beta I(t), \qquad 0<t<t_1. \quad ...(1)$$

$$\frac{d}{dt}I(t) = -\theta I(t)-\frac{\alpha+\beta I(t)}{1+e(T-t)}, \qquad t_1\le t\le t_2 \quad ...(2)$$

$$\frac{d}{dt}I(t) = -\frac{\alpha+\beta I(t)}{1+e(T-t)}, \qquad t_2\le t\le T \quad ...(3)$$

And boundary conditions are $I(0)=q$, $I(t_2)=0$.
Now under the boundary conditions, solutions of differential equations (1), (2), (3) are given as

$$I(t) = -\frac{\alpha}{\beta}+\left(q+\frac{\alpha}{\beta}\right)e^{-\beta t} \qquad ...(4)$$

$$\begin{aligned} I(t) = \alpha\Big\{&(1-\beta T-et)(t_2-t) \\ &+\frac{1}{2}(\beta+e-\beta\theta_0 t-e\theta_0 t+\theta_0)\left(t_2^2-t_2^2\right) \\ &+\frac{1}{3}\left(\beta\theta_0+e\theta_0-\frac{\beta\gamma T}{2}-\frac{e\gamma T}{2}+\frac{\gamma}{2}\right)\left(t_2^3-t^3\right) \\ &+\frac{\gamma}{8}(\beta+e)\left(t_2^4-t^4\right)-\left(\beta+\theta_0-\beta e\theta_0-\theta_0 eT\right. \\ &\left.-\beta e\theta_0 T^2\right)\left(tt_2-t^2\right)-\frac{1}{2}\left(\beta e+2\beta\theta_0+e\theta_0\right) \\ &\times\left(tt_2^2-t^3\right)-\frac{1}{6}\left(2\beta e\theta_0-\beta e\gamma T+\beta\gamma+\theta_0\gamma\right. \\ &\left.-e\gamma\theta_0 T-\theta_0\beta e\gamma T^2\right)\left(tt_2^3-t^4\right) \\ &-\frac{\gamma}{8}(\beta e+\beta\theta_0+\theta_0 e+\theta_0\beta eT)\left(tt_2^4-t^5\right) \\ +\frac{1}{2}&\left(2\beta\theta_0-2\beta\theta_0 eT-\gamma+\gamma eT+\gamma\beta eT^2\right)\left(t^2t_2-t^3\right) \\ +\frac{1}{4}&\left(2\beta e\theta_0-\beta\gamma-e\gamma-\gamma\theta_0+e\gamma\theta_0 T-\gamma\beta eT\right. \\ &\left.+\gamma\beta eT\theta_0\right)\left(t^2t_2^2-t^4\right)-\frac{\gamma e\theta_0}{6}(2\beta T+1)\left(t^2t_2^3-t^5\right) \\ &+\frac{\gamma\beta e\theta_0}{2}\left(t^2t_2^4-t^6\right)+\frac{\beta\gamma t^3}{2}(1-eT)(t_2-t) \\ &+\frac{\beta\gamma t^3}{4}\left(e-e\theta_0 T+\theta_0\right)\left(t_2^2-t^2\right)+\frac{\beta\gamma e\theta_0 t^3}{6}\left(t_2^3-t^3\right)\Big\} \end{aligned} \quad ..(5)$$

$$I(t) = -\frac{\alpha}{\beta}\left[1-\left\{\frac{1+e(T-t)}{1+e(T-t_2)}\right\}^{\beta/e}\right] \qquad ...(6)$$

Now the total cost $C(t_1,t_2,T)$ per cycle time T is

$C(t_1,t_2,T) = \frac{1}{T}$ [Ordering Cost +Holding Cost +Shortage Cost +Opportunity Cost +Purchasing Cost]

Where

Ordering Cost = A

$$\begin{aligned} \text{Holding Cost} = C_h\Bigg[&\frac{1}{\beta}\left\{\left(q+\frac{\alpha}{\beta}\right)\left(1-e^{-\beta t_1}\right)-\alpha t_1\right\} \\ &+\alpha\left\{\frac{1-\beta T-eT}{2}(t_2-t_1)^2\right. \\ &+\frac{\beta+e+\theta_0-\beta\theta_0 T-e\theta_0 T}{6}\times\left(2t_2^3-3t_2^2+t_1^3\right) \\ &+\frac{2\theta_0(\beta+e)+\gamma(1-\beta T-eT)}{24}\left(3t_2^4-4t_2^3t_1+t_1^4\right) \\ &+\frac{\gamma(\beta+e)}{40}\left(4t_2^5-5t_2^4+t_1^5\right)-\{\beta+\theta_0-e\theta_0(\beta+ \\ &T+\beta T^2)\}\frac{\left(t_2^3-3t_2t_1^2+2t_1^3\right)}{6} \\ &-\frac{\beta e+2\beta\theta_0+e\theta_0}{8}\left(t_2^4-2t_2^2t_1^2+t_1^4\right) \\ &-\frac{\beta e(2\theta_0-\gamma T)+\gamma(\beta+\theta_0)-e\gamma\theta_0 T(1+\beta T)}{60} \\ &\times\left(3t_2^5-5t_2^3t_1^2+2t_1^5\right) \\ &-\frac{\beta\gamma(e+\theta_0)+\theta_0\gamma e(1+\beta T)}{48}\left(2t_2^6-3t_2^4t_1^2+t_1^6\right) \\ &+\frac{2\theta_0\beta(1-eT)-\gamma+\gamma eT(1+\beta T)}{24}\left(t_2^4-4t_2t_1^3+3t_1^4\right) \\ &+\frac{\theta_0\beta e(2+\gamma T)-\gamma(\beta+e+\theta_0)+\gamma eT(\theta_0-\beta)}{30}\left(2t_2^5-5t_2^2t_1^3+3t_1^5\right) \\ &-\frac{e\theta_0\gamma(2\beta T+1)}{36}\left(t_2^3-t_1^3\right)^2+\frac{\beta\gamma(1-eT)}{40}\left(t_2^5-5t_2t_1^4+4t_1^5\right) \\ &+\frac{\beta\gamma\left(e-e\theta_0 T+\theta_0\right)}{48}\left(t_2^6-3t_2^2t_1^4+2t_1^6\right) \\ &\left.+\frac{\beta\lambda e\theta_0}{24}\left(t_2^7-t_2^3t_1^3(t_1+t_2)-t_1^7\right)\right\}\Bigg] \end{aligned}$$

Shortage Cost = $-\frac{C_s\alpha}{\beta}(T-t_2)(1+e)$.

Opportunity Cost = $R[\alpha(T-t_2)-1]$.

Purchase Cost = $C_p\left[q+\frac{\alpha}{\beta}\right]$.

Using above values in equation (7), we get the total cost per cycle.

$$C(t_1,t_2,T)=\frac{A}{T}+C_h\left[\frac{1}{\beta T}\left\{\left(q+\frac{\alpha}{\beta}\right)\right.\right.$$

$$\times\left(1-e^{-\beta t_1}\right)-\alpha t_1\Big\}\quad+\alpha\left\{\frac{1/T-\beta-e}{2}(t_2-t_1)^2\right.$$

$$+\frac{(\beta+e+\theta_0)/T-\beta\theta_0-e\theta_0}{6}\times\left(2t_2^3-3t_2^2+t_1^3\right)$$

$$+\frac{2\theta_0(\beta+e)/T+\gamma(1/T-\beta-e)}{24}\left(3t_2^4-4t_2^3t_1+t_1^4\right)$$

$$+\frac{\gamma(\beta+e)}{40T}\left(4t_2^5-5t_2^4+t_1^5\right)-\{\beta/T+\theta_0/T$$

$$-e\theta_0(\beta/T+1+\beta T)\}\frac{\left(t_2^3-3t_2t_1^2+2t_1^3\right)}{6}$$

$$-\frac{\beta e+2\beta\theta_0+e\theta_0}{8T}\left(t_2^4-2t_2^2t_1^2+t_1^4\right)$$

$$-\frac{\beta e(2\theta_0/T-\gamma)+\gamma(\beta+\theta_0)/T-e\gamma\theta_0(1+\beta T)}{60}$$

$$\times\left(3t_2^5-5t_2^3t_1^2+2t_1^5\right)$$

$$-\frac{\beta\gamma(e+\theta_0)+\theta_0\gamma e(1+\beta T)}{48T}\left(2t_2^6-3t_2^4t_1^2+t_1^6\right)$$

$$+\frac{2\theta_0\beta(1-eT)-\gamma+\gamma eT(1+\beta T)}{24T}\left(t_2^4-4t_2t_1^3+3t_1^4\right)$$

$$+\frac{\theta_0\beta e(2+\gamma T)-\gamma(\beta+e+\theta_0)+\gamma eT(\theta_0-\beta)}{30T}\left(2t_2^5-5t_2^2t_1^3+3t_1^5\right)$$

$$-\frac{e\theta_0\gamma(2\beta+1/T)}{36}\left(t_2^3-t_1^3\right)^2+\frac{\beta\gamma(1/T-e)}{40}\left(t_2^5-5t_2t_1^4+4t_1^5\right)$$

$$+\frac{\beta\gamma(e/T-e\theta_0+\theta_0/T)}{48}\left(t_2^6-3t_2^2t_1^4+2t_1^6\right)$$

$$\left.\left.+\frac{\beta\lambda e\theta_0}{24T}\left(t_2^7-t_2^3t_1^3(t_1+t_2)-t_1^7\right)\right\}\right]$$

$$-\frac{C_s\alpha}{\beta}(1-t_2/T)(1+e)+\frac{R}{T}\left[\alpha(T-t_2)-1\right]+$$

$$\frac{C_p}{T}\left[q+\frac{\alpha}{\beta}\right]\qquad\text{...(8)}$$

Now our objective is to minimize the total cost. So we find the all first and second order partial derivatives with respect to t_1, t_2 and T. i.e.

$$\frac{\partial C}{\partial t_1},\frac{\partial C}{\partial t_2},\frac{\partial C}{\partial T},\frac{\partial^2 C}{\partial t_1^2},\frac{\partial^2 C}{\partial t_2^2},\frac{\partial^2 C}{\partial T^2},\frac{\partial^2 C}{\partial t_1\partial t_2},\frac{\partial^2 C}{\partial t_1\partial T},\frac{\partial^2 C}{\partial T\partial t_2}$$

Now the total cost is minimum if $\frac{\partial^2 C}{\partial t_1^2}>0$,

$$.\ \begin{vmatrix}\frac{\partial^2 C}{\partial t_1^2} & \frac{\partial^2 C}{\partial t_1\partial t_2}\\ \frac{\partial^2 C}{\partial t_2\partial t_1} & \frac{\partial^2 C}{\partial t_2^2}\end{vmatrix}>0 \text{ and } \begin{vmatrix}\frac{\partial^2 C}{\partial t_1^2} & \frac{\partial^2 C}{\partial t_1\partial t_2} & \frac{\partial^2 C}{\partial t_1\partial T}\\ \frac{\partial^2 C}{\partial t_2\partial t_1} & \frac{\partial^2 C}{\partial t_2^2} & \frac{\partial^2 C}{\partial t_2\partial T}\\ \frac{\partial^2 C}{\partial T\partial t_1} & \frac{\partial^2 C}{\partial T\partial t_2} & \frac{\partial^2 C}{\partial T^2}\end{vmatrix}>0$$

IV. NUMERICAL EXAMPLE

Now, we present a numerical example.

By the result in equation (8), if we take some parameters A = 250 Rs, C_s = 5 Rs, C_p = 25 Rs, q = 1000 units, R = 2 Rs, C_h = 0.1 Rs as constant values.

e	α	β	γ	θ_0	t_1	t_2	T	$C(t_1,t_2,T)$
150	0.1	40	0.5	0.1	0.199	1.7374	3.0032	8524.49
160	0.1	40	0.5	0.1	0.203	1.7148	2.98	8587.02
170	0.1	40	0.5	0.1	0.209	1.6922	2.9568	8650.05
180	0.1	40	0.5	0.1	0.211	1.6696	2.9336	8713.67
190	0.1	40	0.5	0.1	0.215	1.647	2.9104	8778.03
150	0.2	40	0.5	0.1	0.211	1.509	2.78	9139.41
150	0.3	40	0.5	0.1	0.246	1.385	2.695	9437.05
150	0.4	40	0.5	0.1	0.251	1.314	2.614	9719.51
150	0.5	40	0.5	0.1	0.255	1.26	2.576	9854.98
150	0.1	38	0.5	0.1	0.191	1.756	3.0104	8507.18
150	0.1	36	0.5	0.1	0.187	1.776	3.028	8461.24
150	0.1	34	0.5	0.1	0.181	1.795	3.0386	8434.22
150	0.1	32	0.5	0.1	0.175	1.799	3.0542	8386.01
150	0.1	40	0.6	0.1	0.186	1.733	2.9556	8673.12
150	0.1	40	0.7	0.1	0.173	1.7245	2.8821	8930.43
150	0.1	40	0.8	0.1	0.159	1.7008	2.8212	9131.72
150	0.1	40	0.9	0.1	0.146	1.6849	2.7606	9343.60
150	0.1	40	0.5	0.2	0.178	1.76	2.75	9399.15
150	0.1	40	0.5	0.3	0.161	1.771	2.545	10256.46
150	0.1	40	0.5	0.4	0.138	1.779	2.349	11225.81
150	0.1	40	0.5	0.5	0.118	1.799	2.199	12153.32

CONCLUSIONS

In the present model we develop an optimum order quantity inventory model for perishable items with inventory dependent demand rate. Shortages in inventory are allowed also. Deterioration of items is permissible. In the model the total cost depends on different factors and cost decreases as decreasing value of e, α, β, γ, θ_0. The effects in total cost due to above parameter are given in table as well as graphs. The total cost is very sensitive form these parameters. Increasing value of 'e' increase the time t_1, total cost C and decrease time t_2, T. Increasing value of 'α' increase the time t_1, total cost C and decrease time t_2, T. Decreasing value of 'β' increase the time t_2, T and decrease time t_1, total cost C. Increasing value of 'γ' increase the total cost and decrease time t_1, t_2, T. Increasing value of 'θ_0' increase the time t_2, total cost and decrease time t_1, T. In present model we use deterioration, demand, stock level, holding cost, opportunity cost and deterioration cost. In practical life situation there are many other factors which are responsible to change the total cost. This model can extended with other effective factors and it could be done in future research.

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